Choosing your battles: endogenous multihoming and platform competition*

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June 1, 2023

Abstract

We study how digital platforms can choose competitive strategies to influence the number of multihoming consumers. Platforms compete for consumers and advertisers. A platform earns a premium from advertising to singlehomers, as it is a gatekeeper to these consumers. Competitive strategies leading to intense competition on the consumer side reduce profits on that side, but also increase consumer singlehoming and hence market power over advertisers. The size of the singlehoming premium determines where this competitive strategy 'seesaw' will end up. We apply this insight to four strategic choices that may affect singlehoming: reducing product differentiation, portfolio diversification, the choice of compatibility and tying.

Keywords: Two-sided markets; Internet platforms; Competitive strategy; Singlehoming; Multihoming

JEL classification: D43, L13, L41, L82, L86, M37

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*The authors thank seminar participants at the Universities of Groningen, Louvain-la-Neuve, Tilburg, and the National University of Singapore, at the Netherlands Authority for Consumers and Markets, at the Mannheim/ZEW conference on ICT, EARIE 2021, and the 2021 Oligo Workshop, as well as Markus Reisinger and Tat-How Teh for useful comments.

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1 Introduction

Big techs started out as very distinct firms. Google used to be a simple search engine. Amazon started as an online book seller, Facebook as a social network, and Apple as a computer manufacturer. But increasingly these firms are developing full digital ecosystems that compete head-on for consumers, as also noted in a recent leader in the Economist.[1] Almost all now offer competing cloud computing services, home assistants, and media distribution platforms. Amazon is considered a major threat to Google's targeted ad services, using its detailed consumer data to help others find their audiences.[2] Facebook recently launched "Facebook Shops" that will use its social network data to direct consumers to online stores, challenging for example Amazon and Ebay.[3] Google's Youtube explores selling products through its videos.[4]

From a competition perspective it is puzzling why these firms choose to compete head-on for consumers rather than just enjoying a monopoly within their original niche. In this paper, we explore this question. We argue that platforms can choose on which side of the market to compete. By choosing strategies leading to head-on competition on the consumer market, platforms can induce consumers to singlehome rather than to spend time on many different platforms. This may increase competition for these consumers – and reduce prices on that side of the market – but it also gives the platform more market power on the other side of the market where they sell consumer attention or data to advertisers.[5] When consumers singlehome on a platform, advertisers that want to reach those consumers

[1] As the Economist describes,"First, the companies are increasingly selling the same products or services. Second, they are providing similar products and services on the back of different business models, for example giving away things that a rival charges for (or vice versa, charging for a service that a competitor offers in exchange for user data sold to advertisers). Third, they are eyeing the same nascent markets, such as artificial intelligence (AI) or self-driving cars." (from: The new rules of competition in the technology industry, The Economist, Feb 27th 2021).

[2] see e.g. "Amazon Knows What You Buy. And It's Building a Big Ad Business From It." by Karen Weise in the New York Times, Jan. 20 2019.

[3] "Facebook takes on Amazon with online shopping venture", Financial Times, May 2020

[4] "YouTube explores selling products to take on Amazon", competitionpolicyinternational.com, Oct 2020.

[5] More generally, we are thinking about advertising slots, product referrals, targeting data, etc. We will refer to this as advertising, with the implicit understanding that we have in mind broader applications than merely showing an ad on one's website.

have no choice but to go through that platform. The platform then acts as a gatekeeper, and can charge monopoly access fees to advertisers (Armstrong, 2006; Armstrong and Wright, 2007). This ability to monopolize the advertiser side may outweigh the loss in market power on the consumer side. Platforms can thus choose their battles: they can choose where to compete fiercely in order to gain market power on the other side of the market. We study a number of applications: product positioning compatibility and tying. In all those applications, we show that our approach generates novel insights.

In our platform model, consumers can multihome. Most papers in this literature assume that the number of multihoming consumers is exogenously given (Ambrus et al., 2016; Anderson et al., 2017). Following Gentzkow (2007), we allow consumers to endogenously decide whether to single- or multihome. This is crucial for our argument. By their choice of competitive strategy, platforms affect consumers' homing choice, which in turn affects the platforms' profits on the advertising market.

Our argument is fundamentally different from the well-known point that platforms may charge a low price on one side of the market to get the other side on board (see e.g. Rochet and Tirole, 2003). In our model, competing head-on for consumers implies having more market power on the advertising side. Hence our argument is about ex-ante strategic choices, influencing consumers' homing patterns in order to exploit market power on the other side, rather than about exploiting cross-platform network externalities, taking those strategic choices as given (as in Armstrong and Wright, 2007, for example).

In our general framework, two firms compete in prices in a Perloff-Salop (1985) type model. They face a non-negative price constraint (as in e.g. Choi and Jeon, 2021). Patterns of substitutability may vary. At one extreme, products may be completely independent, so the choice to buy one product is independent of whether the other product is bought. At the other extreme, products may be substitutes, so consuming multiple products has no advantages to a consumer whatsoever over and above consuming just one. Take newspapers for example; a consumer may be perfectly happy to buy a sports paper and a general interest newspaper, but is unlikely to consume two sports papers as these will largely carry the same news. Head-on competition (say, both firms producing a sports paper) reduces each platform's consumer base, but also leads to less sharing of consumers with rivals. A platform will have higher advertising revenues from every consumer that single- rather than multihomes at its platform. We will refer to these additional advertising revenues as the *singlehoming rent*. These rents may outweigh the loss of revenues from fiercer competition on the consumer-side of the market.

As noted, we discuss a number of applications of our framework. First, consider a duopoly of single-product firms. When the singlehoming rent is large, firms are more likely to offer close substitutes rather than to evade competition by creating products that are strongly differentiated. If products are closer substitutes, competition is more intense, lowering demand for each firm. But it also implies that fewer consumers buy both products, increasing the number of singlehoming rents that can be earned from advertisers. The non-negative pricing constraint makes sure that not all these rents will be passed through to consumers. When singlehoming rents are large, these more than compensate the loss from lower consumer prices.

Second, we study multi-product duopolies. We allow firms to either specialize by offering two similar products, or to diversify by offering two unrelated products. There now is a trade-off between earning more on the consumer side when specializing, or earning more on the advertiser side when diversifying. The latter would induce a larger fraction of consumers to singlehome – and hence would increase the number of singlehoming rents earned.

Third, we look at the choice of compatibility. In a two-product duopoly each firm produces its own versions of two complementary products. They can choose to make all products compatible allowing consumers to mix-and-match, or to make them incompatible, thereby forcing consumers to buy two complementary products from the same firm. Incompatibility leads to more intense competition and hence lower consumer prices (see Matutes and Regibeau, 1992).[6] But it also leads to singlehoming. Hence, when singlehoming rents are sufficiently high, firms do prefer incompatibility, and take the resulting fiercer

[6] Zhou (2017) finds that this conclusion can be invalidated if more than two firms compete.

competition on the consumer side in their stride.[7]

Fourth, we study tying. A monopolist producing two complementary products (say, an operating system and an app) faces entry from a firm selling one of these products (a rival app). We study the monopolist's tying strategy, building on Choi and Jeon (2021),[8] and show that with singlehoming rents, total industry profits can be higher under tying than if the monopolist allows more efficient competing apps to run on its system. Even when allowing for side payments, tying is then the optimal strategy. We thus provide a novel theory of harm, based on the difference between single- and multihoming rents, for cases such as the EC's Google Android case.

Our work builds on the literature on platforms, pioneered in e.g. Caillaud and Jullien (2003), Rochet and Tirole (2003), Armstrong (2006) and Armstrong and Wright (2007). Some recent work studies single- versus multihoming in media markets. Ambrus et al. (2016) and Athey et al. (2018) explore the effects of multihoming on advertising intensity. Anderson et al. (2017) point out how singlehoming premiums can be understood through incremental pricing, while Anderson et al. (2019) explore equilibrium entry in a consumer multihoming setting in a Vickrey-Salop circle model. Gentzkow et al. (2022) provide empirical evidence for such singlehoming premiums. Our contribution to this literature is to endogenize consumers' preference for multihoming by allowing platforms to make strategic decisions that affect consumer homing choices in the subsequent pricing subgame. Choi (2010) (see also Choi et al., 2017) is an early paper that considers implications of multihoming from tying decisions of competing platforms. In their paper, consumers' platform choice is determined by content providers. We instead focus on strategic choices made by platforms themselves, where consumers get no direct utility from the other side of the market, a set-up more readily applicable to tech platforms.

Many early papers focus on effects on fee structure, identifying "seesaws" in platform

[7]Athey and Scott-Morton (2021) relatedly study such interoperability decisions by platforms, and refer to the creation of such non-interoperable platforms, in order to decrease multihoming, as platform annexation.

[8]See also Amelio and Jullien (2012) for an early analysis of tying in the context of a zero-pricing constraint.

pricing structure (Rochet and Tirole, 2006), where a platform may reduce price on one side of the market to be able to attract more higher-margin consumers on the other side. Belleflamme and Peitz (2019) and Liu et al. (2020) explore which side of the platform benefits when the extent of multihoming changes exogenously on either side. Our paper argues that platforms may also face a seesaw in the competitive strategies that precede the pricing stage: softening competition for consumers induces consumers to multihome, which intensifies competition for advertisers.[9]

Other related literature includes the following. In Klemperer (1992) consumers face shopping costs, and multi-product firms choose minimum differentiation and compete head-on to avoid that consumers also visit their competitor. In Gabszewicz et al. (2004), free-to-air media firms compete in advertising time to attract ad-avoiding consumers. With minimum differentiation, cutthroat competition forces the amount of advertising to zero, hence media firms cannot make any money from ads on their platform. In Gal-Or and Dukes (2003) however, advertising serves to inform consumers, so less advertising leads to more market power for advertisers, hence media firms can charge higher advertising prices. This gives them an incentive for minimum differentiation. Peitz and Valletti (2008) compare pay-tv versus advertised financed free-to-air television, and finds that in the latter, content differentiation decreases with consumers' ad nuisance. Our contribution to this literature is the focus on singlehoming rents from advertisers that may result from more intense competition on the platforms' consumer side.

We do not address the total welfare effects of the firms' strategies we study. This would require, among other things, more detail on the advertising side, as e.g. Prat and Valletti (2022) who consider implications for competition between advertisers in their sales to ultimate consumers.

The remainder of this paper is structured as follows. In Section 2, we present our general framework. The choice of platform content is analyzed in Section 3, with Section 3.1 focusing on single-product firms, while Section 3.2 studies multi-product firms.

[9] Anderson and Peitz (2020) refer to a "seesaw" involving the extent to which interests of advertisers and consumers are aligned.

Compatibility is discussed in Section 4, and we consider tying in Section 5. Section 6 concludes.

2 The General Framework

We consider a two-stage model. In stage 1, firms make strategic choices on their product portfolios. In stage 2, they compete on the product market. Firms make money not only from selling their products to consumers, but also from gaining access to these consumers' eyeballs or data. For example, they may show advertisements to their users. They may also sell consumer data to potential advertisers for targeting purposes, or use this data to improve their recommendation systems. Either way, having access to consumer data and eyeballs is valuable for a firm. For ease of exposition, in the remainder of the paper we simply refer to this as advertising. The value of this advertising depends on whether consumers only buy from one firm (singlehoming) or buy from multiple firms (multihoming). In the case of singlehoming, a firm is a monopoly gatekeeper to its consumer's eyeballs. With multihoming, some of the gatekeeping rents are dissipated.

Consumers We model consumers in Perloff-Salop (1985) fashion. A unit mass of consumers differ in their valuation for each product $i = 1, \ldots, n$. Consider one representative consumer. Her stand-alone utility from consuming i is denoted v_i, and is a random draw from some distribution F on $[0, 1]$. For simplicity, we assume that valuations for all products are independent draws from the same distribution. We assume F is smooth, and density f is strictly positive on $[0, 1]$.

The price of product i is denoted P_i. We assume that firms can only charge non-negative prices, as abuse by consumers of negative prices would be hard to police. Hence, $P_i \geq 0$. Consumers have unit demand for each individual product. However, a consumer may consume more than one product. If she does, her total utility may differ from the sum of the individual utilities of all products she buys: products may be (partial) substitutes. When reading two newspapers for example, there will be some overlap in the news that they cover. Also, when subscribing to a second video streaming service, most people will not double their time spent watching TV – nor their enjoyment in doing so. In other words, the total utility a consumer derives from consuming i and j will be lower than the sum of the willingnesses-to-pay for these two products when consumed in isolation. Following Gentzkow (2007), we simply assume that total utility will shift downwards by some constant σ_{ij} when i and j are both consumed.

More precisely, for each product i, let I_i be a dummy that denotes whether this consumer buys product i. Thus $I_i = 1$ if she buys i, and $I_i = 0$ if not. We then assume that total utility is given by

$$u = \sum_{i=1}^{n} \left((v_i - P_i)\, I_i - \sum_{j>i}^{n} \sigma_{ij} I_i I_j \right).$$

In this equality, $\sigma_{ij} \in [0, 1]$ represents the degree of substitutability between i and j. Suppose that $\sigma_{ij} = 0$. Then the total utility of consuming i and j simply equals the sum of their individual utilities. The two products are completely independent of each other: consuming one does not compromise the utility obtained from consuming the other. However, if $\sigma_{ij} = 1$, then consuming both products does not add value to the utility of consuming just the better one of the two.[10] In that sense, i and j are now substitutes. This formulation generalizes Gentzkow (2007), who assumes that each two products have the same σ.

Advertising If a consumer uses the products of only one firm, then that firm has monopoly power over access to that consumer's data or eyeballs. If the consumer buys products from multiple firms, then those firms compete in selling access to the consumer. Such competition can be modelled in numerous ways. We simply assume that a firm can earn an amount r_s in advertisement revenues from each of its singlehoming consumers. A consumer that multihomes generates advertising revenues r_m for that firm, with $r_m \leq r_s$,

[10]Consuming only i, for example, would yield $u = v_i - P_i$, while consuming both i and j would yield $u = v_i - P_i + v_j - P_j - \sigma_{ij}$. With $\sigma_{ij} = 1$, $v_j \leq 1$ and non-negative prices, this is always weakly lower.

reflecting that there is a premium to having a monopoly on access to a consumer. This reduced form captures many advertising models. For example, if advertisers only value a single exposure to any consumer and firms compete for advertisers in Bertrand fashion, we simply have $r_m = 0$: the firms compete away all access rents. If a second exposure has value, we have $r_m > 0$, with each firm being able to only capture the incremental rents of the second exposure, as in Anderson et al. (2017). [11] More precisely, suppose that a_1 reflects the willingness-to-pay of advertisers for a first exposure, while a_2 reflects that for a second exposure.[12]

Suppose that each firm has n_s single-homing, and n_m multi-homing consumers. An advertiser that only advertises on one platform would be willing to pay $(n_s + n_m)a_1$ for that privilege. If it also advertises on the second platform, it would be willing to pay $n_s a_1 + n_m a_2$ for that – as the multihomers have already been reached on the other platform. If the platforms compete in Bertrand fashion, the price of advertising will thus be competed down to $n_s a_1 + n_m a_2$, so we have $r_s = a_1, r_m = a_2$. Hence, singlehomers command a premium $a_1 - a_2$ in advertising income.

For simplicity, we assume that there is no disutility to consumers from watching advertisements: allowing for that would not affect our qualitative results.

Firms Marginal costs of production for each product are constant and equal to c. Every sale a firm makes also yields an advertising revenue of (at least) r_m. Define R as the monopoly premium a firm earns from controlling unique access to a consumer's eyeballs: $R \equiv r_s - r_m$. We refer to R as the *singlehoming premium.* It captures the incremental value highlighted in Anderson et al. (2017). For ease of exposition, we will work with the net marginal cost defined as $C \equiv c - r_m$. We may very well have $C < 0$, if (multihoming) advertising benefits outweigh production costs. A firm that sells product i to a multihoming

[11] Gentzkow et al. (2022) generalize the framework in Anderson et al. (2017) and also provide additional examples where it applies.

[12] If we assume that advertising on one product yields one and only one impression. For example, suppose that an advertiser is willing to pay an amount A for each consumer that sees its ad, but also that each ad is only seen by a given consumer with probability ϕ. The advertiser would thus be willing to pay $a_1 = \phi A$ for the first ad, but only $a_2 = (1 - \phi)\phi A$ for the second – as $(1 - \phi)\phi$ is the probability that a consumer sees the second ad without having seen the first.

consumer thus has net marginal costs C and revenues P_i. If it sells i to a singlehoming consumer, it has net marginal costs C and revenues $P_i + R$.

Firms play a two-stage game. First, they make strategic decisions regarding their product portfolio. Second, they compete in prices. In the price-setting stage, firms maximize profits that consist of the margin $P_i - C$ on each product sold, plus the premium R for each consumer that singlehomes at that firm.

We are particularly interested in the competitive strategies firms choose in stage 1. We look at four examples. In our first application, firms make choices regarding product substitutability. In the second, firms choose whether to diversify their portfolio. The firms have to choose whether to specialize into selling substitute products, or to diversify their product portfolio. The third application has firms making compatibility choices. In the final application, a monopoly firm decides whether to tie an app that could be supplied competitively to a system on which the firm is the monopolist producer. In each application, firms make choices that affect their product portfolios. Given the resulting product portfolios as reflected in the ownership structure and substitutability matrix σ, the pricing subgame ensues.

3 Application 1: choice of platform content

3.1 Single-product firms

As a first application, we consider competition between two single-product firms that first position their product by choosing the content of their platform, and then compete in prices. Product positioning is captured by our measure of product substitutability σ_{12}. For simplicity, we refer to this as σ in the remainder of this application. For example, consider two news sites that have to decide what type of news to run. They may each choose to run general news. In that case σ is close to 1. Alternatively, one may run general news while the other focuses on sports. In that case σ is close to zero.

The timing is as follows:

1. Firm 1 enters the market and chooses its content.
2. Firm 2 enters and chooses its content relative to firm 1, as measured by σ.
3. After having observed σ, firms simultaneously and noncooperatively set non-negative prices P_1 and P_2.
4. Consumers make their purchasing decisions and ad revenues are realized.

We first study the subgames with $\sigma = 0$ (independent products) and $\sigma = 1$ (substitutes). We relegate the analysis of intermediate choices of σ to an appendix. We focus on symmetric equilibria in the subsequent pricing games.[13] The equilibrium positioning choice will then simply be the choice that maximizes total profits, making the assumption that it is firm 2 that chooses σ inconsequential.[14]

Analysis With $\sigma = 1$ platforms are substitutes and it makes no sense to multihome: doing so always lowers net utility. A consumer prefers platform 1 whenever

$$\begin{aligned} v_1 - P_1 &\geq v_2 - P_2 \\ v_1 - P_1 &\geq 0 \end{aligned}$$

This is area 1 in Figure 1 – which we have drawn for the case that $P_1 > P_2$. In the Figure, consumers in area 2 buy from firm 2, while consumers in area $\emptyset$ refrain from consumption.

Consumer singlehoming implies that platforms are effectively monopolists on the market for advertisements: as there is no multihoming they simply earn the singlehoming premium R in ad revenues for each consumer that they attract. To derive equilibrium prices, assume

[13] Throughout the paper, we assume that the distribution of match values is such that a unique equilibrium of the pricing game always exists.

[14] Alternatively, we could consider simultaneous content choices, resulting in a coordination game, with the same outcome for the equilibrium that maximizes industry profits, but leaving the question how firms would coordinate on this equilibrium.

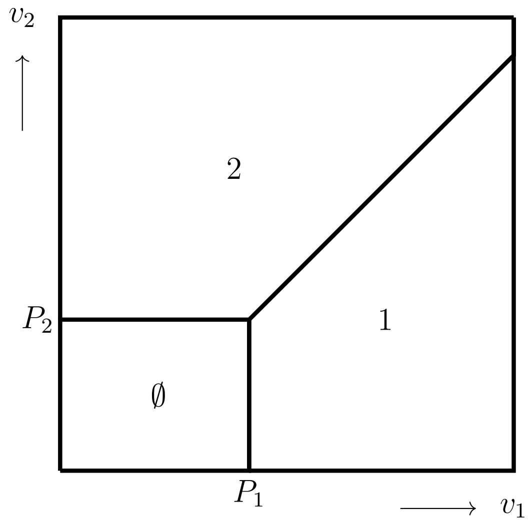

Figure 1: $\sigma = 1$, substitutes

without loss of generality $P_1 \geq P_2$. From Figure 1, total profits of firm 1 are then given by

$$\pi_1 = (P_1 - C + R) \int_{P_1}^{1} F(v_1 + P_2 - P_1) dF(v_1) \tag{1}$$

In a symmetric equilibrium, each firm's profits are

$$\pi^* = (P^* - C + R) \frac{1}{2} \left(1 - F^2(P^*)\right).$$

This is intuitive: firms simply share the part of the market that is covered (i.e. all consumers except for the "zerohoming" consumers in area ∅ in Figure 1, that has size $F^2(P^*)$). P^* is determined by the first-order conditions with respect to P_1 and imposing symmetry:

$$P^* - C + R = \frac{1 - F^2(P^*)}{2f(P^*)F(P^*) + 2\int_{P^*}^{1} f(v)dF(v)},$$

where the left-hand side equals the average mark-up. If this yields negative prices, we have a corner solution with $P^* = 0$.

Next, consider independent products: $\sigma = 0$. In this scenario, platforms are effectively monopolists on the consumer side of the market; their demand does not depend on the

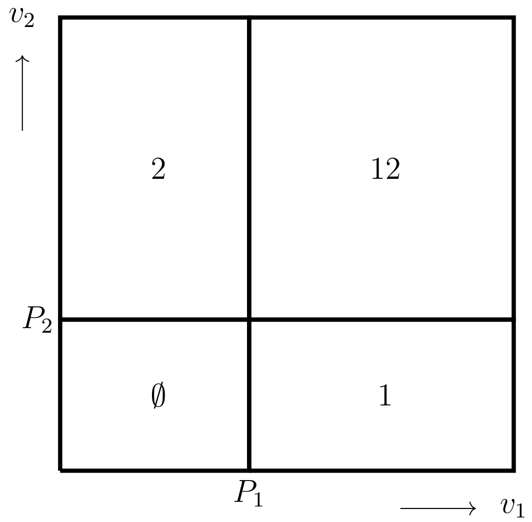

Figure 2: $\sigma = 0$, independent products

price charged by the other firm. Of course, they do now compete on the ad market for advertisements to consumers that multihome. Consumers with $v_1 \geq P_1$ will use platform 1, those with $v_2 \geq P_2$ will use platform 2, those for whom both conditions are satisfied will multihome (see Figure 2).

Profits of platform 1 are given by

$$\pi_1 = (P_1 - C)\,(1 - F(P_1))\,(1 - F(P_2)) + (P_1 - C + R)\,F(P_2)\,(1 - F(P_1))\,,$$

where the first term represents the multihomers (area 12 in Figure 2), while the second represents the singlehomers (area 1 in Figure 2). The firm only earns the advertising monopoly premium R for the latter group.

In a symmetric equilibrium, profits are

$$\pi^* = (P^* - C)\,(1 - F(P^*)) + RF(P^*)\,(1 - F(P^*))\,.$$

where prices P^* satisfy first-order conditions for the symmetric equilibrium,

$$P^* - C + RF(P^*) = \frac{1 - F(P^*)}{f(P^*)}.$$

The left-hand side is the average mark-up, which includes the singlehoming premium R for the fraction $F(P^*)$ of consumers that do not buy the rival's product. The right-hand side is the hazard rate for the distribution of v. As long as net marginal costs C are non-negative (in other words, marginal costs c exceed the advertising revenue made on multihoming consumers r_m), prices will never go to zero: at $P^* = 0$, all consumers would multihome and platforms would make non-positive profits.

A similar analysis for the case of intermediate $0 < \sigma < 1$ is provided in the appendix.

Example For the uniform distribution, $F(v) = v$, and with $C = 0$, the first-order conditions for $\sigma = 1$ can be solved to

$$P^* = \sqrt{2}\sqrt{1 - R} - 1.$$

Note that this solution is only feasible with $R \leq 1/2$: for higher values of R, optimal prices would be negative. As we do not allow for that, we get a corner solution at $P^* = 0$. Equilibrium profits if $\sigma = 1$ and $R \leq 1/2$ equal

$$\pi^* = (1 - R)\left(3 - R - 2\sqrt{2}\sqrt{1 - R}\right).$$

It is easy to see that this is increasing in R. With $R > 1/2$, equilibrium prices are zero, so profits become

$$\pi^* = \frac{1}{2}R.$$

If, instead, firms choose to provide independent products $\sigma = 0$, we find

$$P^* = \frac{1}{2 + R}.$$

In this case of independent products, although prices decrease in R (platforms are more eager to attract consumers as their ad revenues from a consumers increase), they never go down to zero. Total profits equal

$$\pi_1 = \left(\frac{1+R}{2+R}\right)^2,$$

which is also increasing in R.

We now turn to the first stage of the game, the choice of strategy. Figure 3 compares equilibrium profits for the cases $\sigma = 0$ and $\sigma = 1$, as well as a narrow range where an intermediate σ dominates. With $\sigma = 0$, the kink at $R = 1/2$ is caused by the switch from strictly positive prices to prices that equal zero. Beyond $R = 1/2$ the curve becomes steeper: an increase in R is no longer mostly competed away by a decrease in equilibrium prices. From the graph it is clear that platforms prefer to set $\sigma = 1$ if and only if the advertising market is sufficiently important (so R is sufficiently high). The shift from $\sigma = 0$ to $\sigma = 1$ is not completely bang-bang in this example: in a small region, just before the $\sigma = 1$ profits start dominating the $\sigma = 0$ profits, it is profit-maximizing to set σ close to, but not equal to 0.

The qualitative result in this uniform distribution example holds for general distributions $F(v)$. While the details of the transition in equilibrium from $\sigma = 0$ to $\sigma = 1$ will vary according to the characteristics of the distribution $F(v)$, the shift is general:

Proposition 1 *For general distribution $F(v)$, if singlehoming premium R is sufficiently small, platforms maximize profits by differentiating their content as much as possible: $\sigma^* = 0$. For R sufficiently large, profits are maximized with substitutes: $\sigma^* = 1$.*

Proof In Appendix.

With $R = 0$ we have standard profit maximization: firms seek a high margin on a high volume of consumers. They can achieve this by differentiating their product as much as possible from their competitor's. By doing so they establish a monopoly which allows

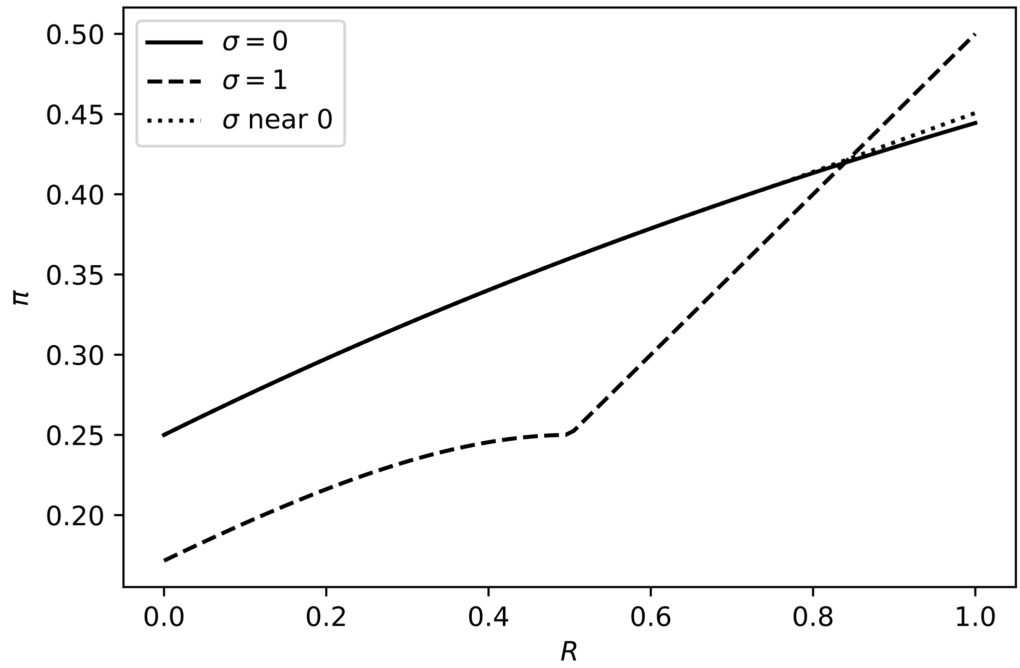

Figure 3: Profits, as a function of singlehoming premium R, for $\sigma = 0, \sigma = 1$, as well as optimal intermediate σ, for the uniform example.

them to charge their monopoly prices without having to worry about their rival's actions. Consumers with a high willingness to pay for both products end up buying both.

When $R > 0$, however, a platform cannot extract singlehoming advertising rents from consumers that also buy the competitor's product. The higher R, the more platforms start focusing on obtaining a high volume of consumers *that are not also served by their rival.* This can be achieved by offering a product that is virtually identical, creating fierce competition for consumers. For large R, the gain in singlehoming advertising rents from offering a substitute, more than offsets the decrease in profits due to fierce competition on the consumer side. The non–negative-pricing constraint is important in this argument: if firms could charge negative prices, higher advertising rents would mostly be competed away by charging ever lower prices to consumers.

Hence, by their choice of σ, platforms can effectively choose to compete head-on for consumers (by producing substitutes and setting $\sigma = 1$) or to compete head-on for advertisers (by producing independent products and setting $\sigma = 0$). If advertising rents from singlehoming are not so important, R is low, and platforms prefer to monopolize the consumer market, while competing vigorously for advertisers. As advertising singlehoming rents become more important however, at some point it becomes more profitable to turn all consumers into singlehomers: by doing so platforms monopolize the advertising market, competing fiercely for consumers instead.

3.2 Multiproduct firms

We now extend the model of the previous subsection to study the case of multiproduct firms. In particular, we explore whether firms want to sell portfolios of similar products, or rather prefer to have diversified portfolios. Suppose for example that firm A is a gaming platform, while firm B is a search engine. Both firms consider extending their product offerings. Firm A could decide to offer a second, differentiated gaming platform, but it could also choose to diversify and offer a competing search engine. Similarly, firm B could choose to offer a second, differentiated search engine, but it can also choose to diversify into gaming, hence competing head-on with firm A. The timing is as follows:

1. Firms either choose to specialize (offering two products of the same type) or to diversify (offering two different products).

2. Firms simultaneously and non-cooperatively set prices for the products they sell.

3. Consumers make their purchasing decisions and ad revenues are realized.

For simplicity and to avoid having to study asymmetric market configurations, we assume that both firms make the same choice in the first stage. Again, firms make money not only from selling to consumers, but also from showing ads or otherwise controlling access to their consumers.

To analyze this model using our framework, consider four products $1, 2, 3$, and 4. Assume that the matrix of substitution parameters $\boldsymbol{\sigma}$ is given by

$$\boldsymbol{\sigma} = \begin{bmatrix} . & 1 & 0 & 0 \\ 1 & . & 0 & 0 \\ 0 & 0 & . & 1 \\ 0 & 0 & 1 & . \end{bmatrix}. \tag{2}$$

Hence, products 1 and 2 are substitutes for each other, as are 3 and 4. But any $i \in \{1, 2\}$ is independent of any $j \in \{3, 4\}$. 1 and 2 could be two gaming platforms, while 3 and 4 could be two search engines. We again assume that $v_i \sim F(v)$. For ease of exposition and to focus on the fact that we are looking at two different product types, we will label these four products as $A1$, $A2$, $B1$ and $B2$ respectively, where both As represent similar product types (e.g., both are gaming platforms), and likewise for both Bs.

Below, we first study the specialization and diversification subgames. We then show our main result in this application: when R is small, firms choose to specialize. If R is large, however, they prefer to diversify.

Analysis First consider the case in which both firms specialize. This leads to competition between an AA and a BB firm. As $A1$ and $A2$ are substitutes, consumers buy at most one of these. In equilibrium, firm AA will set the same price P_A for each of its products and consumers choose the product that gives the highest v_i, if any. As the cumulative distribution of $\max\{v_1, v_2\}$ is given by $F^2(v)$, AA's profits are given by

$$\pi_{AA} = \left(1 - F^2(P_A)\right)\left(P_A - C + RF^2(P_B)\right),$$

with P_B the price that the other firm charges for $B1$ and $B2$. Firm AA collects the singlehoming premium R only for its consumers that buy neither $B1$ nor $B2$, which is a share $F^2(P_B)$.

The analysis is identical to the $\sigma = 0$ case in the previous section, but with the term

F^2 rather than F. The first-order condition for a symmetric interior equilibrium at price P^* is given by

$$P^* - C + RF^2(P^*) = \frac{1 - F^2(P^*)}{2F(P^*) \cdot f(P^*)}. \quad (3)$$

The left-hand side is the average mark-up, including the singlehoming premium for part of the consumers. This mark-up equals the hazard rate for the distribution of the maximum valuation F^2 on the right-hand side. If C is non-negative, the zero-pricing constraint will never bind.

Now consider the diversification scenario. We then have two firms: one selling products $A1$ and $B2$, the other selling $A2$ and $B1$. On the consumer side, for each of the two product types, the analysis is identical to that of $\sigma = 1$ in the previous section. But the advertising side is different. There are now multihoming consumers: those that buy A and B from different firms. Without loss of generality, suppose $P_{A1} \geq P_{A2}$ and $P_{B1} \geq P_{B2}$. Denote the sales of product αi as $Q_{\alpha i}$, with $\alpha, i \in \{A, B\} \times \{1, 2\}$. Similar to (1), we have

$$Q_{\alpha 1} = \int_{P_{\alpha 1}}^{1} F(v_{\alpha 1} - P_{\alpha 1} + P_{\alpha 2}) dF(v_{\alpha 1}), \quad (4)$$

with a similar expression for $Q_{\alpha 2}$.

We derive the profits of firm 1. It earns the singlehoming rent R on three types of consumers. First, there are those that buy $A1$ and do not buy any B. This is a mass $Q_{A1}(1 - Q_{B1} - Q_{B2})$. Second, there are those that buy $B2$ and do not buy any A. This is a mass $Q_{B2}(1 - Q_{A1} - Q_{A2})$. Third, there are those that buy $A1$ and $B2$. This is a mass $Q_{A1}Q_{B2}$. Profits of firm 1 can thus be written

$$\begin{aligned} \pi^1_{AB} &= (P_{A1} - C) Q_{A1} + (P_{B2} - C) Q_{B2} \\ &\quad + R \left[Q_{A1}(1 - Q_{B1} - Q_{B2}) + Q_{B2}(1 - Q_{A1} - Q_{A2}) + Q_{A1}Q_{B2} \right]. \quad (5) \end{aligned}$$

We look for a symmetric equilibrium. We can then write $Q_{A1} = Q_{B1} = Q_{A2} = Q_{B2} \equiv Q^*$, and profits of each firm collapses to

$$\pi_{AB} = 2\left(P^* - C\right)Q^* + R\left[2Q^*\left(1 - 2Q^*\right) + Q^{*2}\right]. \tag{6}$$

From Figure 1 it is easy to see that in an internal symmetric equilibrium, $Q^* = \frac{1}{2}\left(1 - F^2(P^*)\right)$. Equilibrium profits can now be written

$$\pi_{AB} = \left(P^* - C\right)\left(1 - F^2(P^*)\right) + R \cdot \left[\left(1 - F^2(P^*)\right) F^2(P^*) + \frac{1}{4}\left(1 - F^2(P^*)\right)^2\right],$$

where the first term reflects the income from consumers, while the R-dependent terms consist of those consumers that only consume $A1$ or only $B2$ (first term in the square brackets), plus those who consume both $A1$ and $B2$ (second term).

The symmetric equilibrium prices satisfy the first-order condition

$$Q^* = \left(P^* - C + R\left(1 - Q^*\right)\right)\left(F(P^*)f\left(P^*\right) + \int_{P^*}^{1} f\left(v\right)dF\left(v\right)\right) - RF(P^*)f\left(P^*\right)Q^*. \tag{7}$$

Moreover, $\pi_{AB} = -C + \frac{R}{4}$ in case the zero-price constraint binds.

Example For the uniform situation with zero net costs, in the case of specialization the equilibrium condition (3) becomes

$$3P^{*2} + 2RP^{*3} - 1 = 0.$$

For the diversification scenario, we have from (7),

$$2P^* + R(1 + P^{*2} - P^* + P^{*3}) = 1 - P^{*2}.$$

Figure 4 plots the resulting profits. With $R = 0$, we have the standard intuition that each firm cornering its own market preferable for the firms, as it allows for price coordination to fully monopolize that market. When R is large, however, there are large rents to be gained by instead monopolizing the advertising market as much as possible. This is better achieved when firms both engage in diversification, thus creating a larger number of 'one-stop shoppers' that cannot be reached by advertisers through the rival platform. This source of income dominates the loss of income from consumers (who get the products for free as R grows large, with the non-negative price condition explaining the kink in profits).

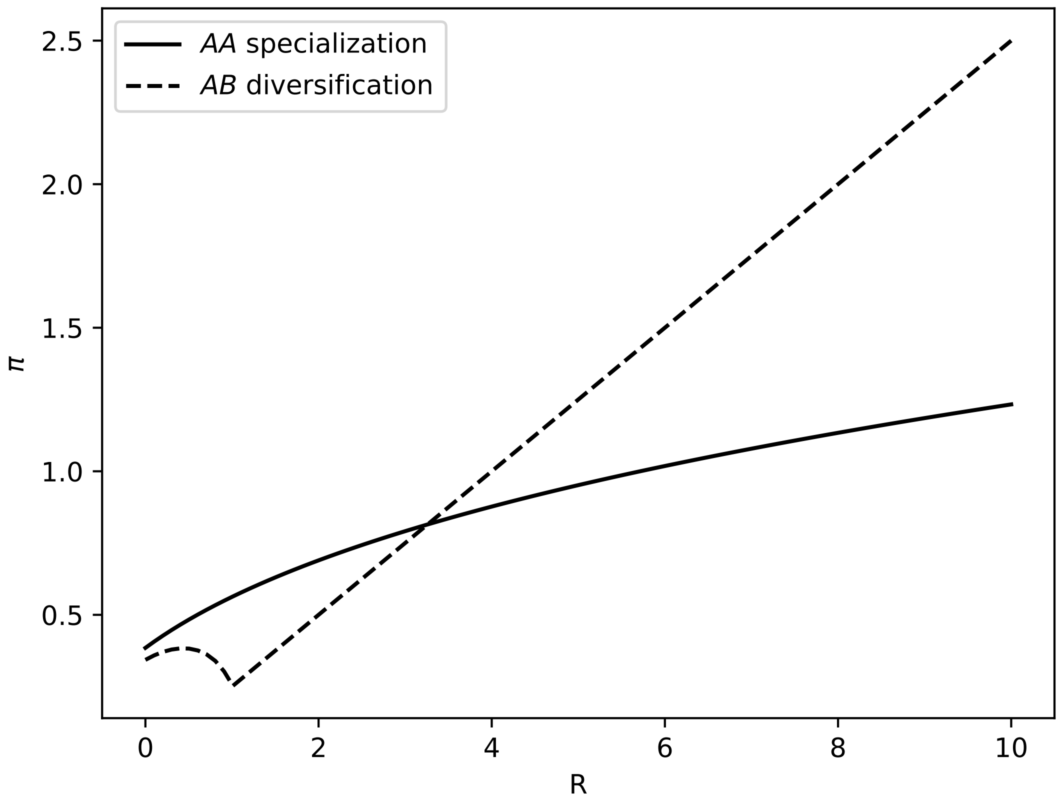

Figure 4: Profits, as a function of singlehoming premium R, for the specialization and diversification scenarios, for the uniform example.

This insight extends to the case with general costs C and distribution F:

Proposition 2 *For general distribution $F(v)$, if singlehoming premium R is small enough, specialization maximizes profits. For R sufficiently large, diversification maximizes profits.*

Proof In Appendix.

There is a strategic see-saw effect that depends on the size of the singlehoming rent R. Firms face a tradeoff between competing on the consumer market and competing on the advertising market. For small R, profits on the consumer market outweigh those on the advertising markets, and firms try to soften competition for consumers. They can do so by specializing. For large R, profits on the advertising markets outweigh those on the consumer markets, and firms try to soften competition for advertisers. They can do so by diversifying, offering unique eyeballs to advertisers and offering a one-stop shop to consumers.

4 Application 2: standardization or incompatibility

Next, we look at compatibility choices for two competing multi-product firms. Suppose firm 1 offers $A1$ and $B1$, while firm 2 offers $A2$ and $B2$. Again, products of the same type are substitutes (so $\sigma_{A1A2} = \sigma_{B1B2} = 1$) while different products offered by the same firm are independent of each other ($\sigma_{A1B1} = \sigma_{A2B2} = 0$). For simplicity, we assume that the markets for both product types are fully covered (see e.g. Zhou, 2017).

Firms can choose whether to make their products compatible by adopting common standards. Consider for example two software producers, both selling a web browser and a video app. Under standardization, firm 1's browser would be able to interface well with 2's video app so $\sigma_{A1B2} = \sigma_{A2B1} = 0$. Under incompatibility, the web browser and video app would be tightly integrated, making it impossible for consumers to mix products from the two firms, even if both products would be offered at zero price. In this case of incompatibility, $\sigma_{A1B2} = \sigma_{A2B1} = 1$. On the advertising side, each firm earns an additional R over those consumers that exclusively buy that firm's products.

The timing is as follows:

1. Firms decide to standardize their products or to make them incompatible.

2. Firms set prices for both products.

3. Consumers make their purchasing decisions and ad revenues are realized.

Note that if one firm chooses to make its products incompatible with their rival's in stage 1, then it de facto forces the other firm to make that same choice.

Analysis First consider the case in which the firms choose to make their products incompatible. By construction, all consumers then singlehome. Suppose that firm 1 charges P_1 for each product in its bundle, so its bundle price is $2P_1$. Firm 2 charges P_2 for each product. Denote $v_i^+ \equiv v_{Ai} + v_{Bi}$, and denote the corresponding density of v^+ as F^+. A consumer will buy a bundle from firm 1 whenever $v_1^+ - 2P_1 > v_2^+ - 2P_2$. Assume without loss of generality that $P_1 \geq P_2$. Profits of firm 1 then equal

$$\pi_1^{\text{incomp}} = (2P_1 - 2C + R) \int_{2P_1-2P_2}^{2} F^+ (v + 2P_2 - 2P_1) \, dF^+(v).$$

As in equilibrium sales per firm equal 1/2, symmetric equilibrium profits are

$$\pi^{\text{incomp}} = P^* - C + \frac{R}{2}.$$

The first-order condition yields[15]

$$2P^* - 2C + R = \frac{1}{2 \int_0^2 f^+(v) dF^+(v)} \tag{8}$$

where the left-hand side is the average markup per bundle. If this yields negative prices, we have a corner solution with $P^* = 0$.

Now consider the case of standardization. Firms then compete separately on each market. Profits from the ad channel connect the two markets: a firm only receives R for consumers that buy both A and B from that firm. Suppose that firm 1 charges P_1 for $A1$ and $B1$, while firm 2 charges P_2 for its products. Again, assume without loss of generality

[15] Note that this is just the analysis with $\sigma = 1$ in section 3, but with a bundle of two products.

that $P_1 \geq P_2$. On each market, firm 1 then sells Q_1 units, with

$$Q_1 = \int_{P_1 - P_2}^{1} F(v + P_2 - P_1) dF(v),$$

Since valuations are independent, profits of firm 1 equal

$$\pi_1^{\text{stand}} = 2(P_1 - C)Q_1 + RQ_1^2,$$

where Q_1^2 is the share of consumers that buy both products from firm 1. Equilibrium profits are

$$\pi^{\text{stand}} = P^* - C + \frac{R}{4},$$

where the first-order conditions for an interior symmetric equilibrium now simplify to

$$2P^* - 2C + R = \frac{1}{\int_0^1 f(v) dF(v)}. \tag{9}$$

Example For the uniform situation with zero net costs, first note that

$$f^+(v) = \begin{cases} v & \text{for } v < 1 \\ 2 - v & \text{for } v > 1, \end{cases}$$

hence $\int_0^2 f^+ dF^+(v) = \int_0^2 f^+(v)^2 \, dv = 2/3$, which implies from (8) that the equilibrium markup for the combined goods is $2P^* + R = \frac{3}{4}$ and profits are $\pi^{\text{incomp}} = \frac{3}{8}$.

With standardization, we immediately have from (9) that $P^* + \frac{1}{2}R = \frac{1}{2}$, so total profits are $\frac{1}{2} - \frac{1}{4}R$. Hence, with $R = 0$, the firms prefer standardization, allowing mix-and-match purchases. Incompatibility leads to fiercer competition for consumers to either buy all or nothing at a particular firm. Standardization thus yields higher prices. For $R > 1/2$ however, firms prefer to make products incompatible. All their clients then become one-stop shoppers that cannot be reached by advertisers through the rival firm. This insight extends to general costs C and distribution F:

Proposition 3 *For a general distribution $F(v)$, if R is sufficiently small, firms choose to offer standardized products in a two-product duopoly. For R sufficiently large, firms choose to make products incompatible.*

With $R = 0$ standardization dominates, as incompatibility intensifies competition. Since the distribution of the average of two i.i.d. variables is more strongly peaked than that of each individual variable, there is a larger density of marginal consumers. For positive R, firms lose advertising income from the mix-and-match consumers. Advertisers can reach those consumers through either firm, and competition for advertisers drives down ad revenues. Instead, with incompatibility, firms monopolize access to consumers, as they provide a one-stop shop. In their choice whether to standardize, firms thus trade off the loss from lower prices against the extra profits from monopolizing advertising to their consumers.

5 Application 3: tying

As a final application, we consider a model of tying. This is a variation on the standardization model in the previous section. As an example, the EC's Google Android case (2018) involves tying of Google's search engine with Google's app store on smartphones. The concern is that Google's app store is essential for Android phone manufacturers; by obliging these manufacturers to also install Google search as the default search engine, Google might foreclose access to end users for rival search engine producers.[16]

An important difference with our earlier analyses is that contracts between app producers (including Google) and phone manufacturers may also include side payments. We therefore include the possibility of transfers between the monopolist and the entrant into our model. In practice, these would be mediated through bilateral contracts with phone manufacturers.

In line with much of the literature on tying, we consider a setting with homogeneous

[16]For a discussion of the details of this case, see Choi and Jeon (2021) and Etro and Caffarra (2017).

consumers, rather than one in which valuations are distributed according to some distribution F. Hence, this is not a direct application of our general framework in Section 2: in this monopolistic setting, we are able to make our point in a much simpler model.

Consider the following set-up (following, e.g., Rey and Tirole, 2007). A monopolist provides a 'system' A, for which it is the only producer, and an 'app' $B1$. In the app market, there are competitive rival app producers that produce their own version $B2$ of that app. A unit mass of consumers value A at v_A, $B1$ at v_{B1}, and, for simplicity, all rival $B2$s at v_{B2}. Moreover, they prefer any rival app $B2$ to $B1$, so $v_{B2} > v_{B1}$. Denote $\Delta \equiv v_{B2} - v_{B1}$. To be able to use either $B1$ or one of the $B2$, consumers need system A. Consumers buy at most one app. For simplicity, marginal costs for all products are zero.

As a benchmark, suppose there is no advertising. First, consider the case without tying. The superior apps $B2$ will enter the market and compete prices down to marginal costs. The equilibrium has $p_{B2} = 0$, and $p_A = v_A + v_{B2} = v_A + v_{B1} + \Delta$; since system A is indispensable for using the app, the monopolist can capture the additional rents created by the rival apps through a higher price for the system. In equilibrium, consumers buy A and one of the $B2$, leaving the monopolist with profits $\pi_A = v_A + v_{B1} + \Delta$. By tying the sale of A to $B1$, the monopolist would do worse. It can then charge at most $p = v_A + v_{B1}$ for the combined system and app. This is the Chicago critique.

As noted by Choi and Jeon (2021), this argument breaks down when there is advertising and prices are restricted to be non-negative. Let us first assume that all ad impressions are equally valuable. In line with our notation in Section 2, suppose that r_m can be earned from ads, both on A and B. We thus assume that there can also be advertising on the system unrelated to $B1$ or $B2$, perhaps through other apps provided by the monopolist.[17]

With tying, the monopolist can now charge $p = v_A + v_{B1}$ yielding profits $\pi_A = v_A + v_{B1} + 2r_m$, as it earns advertising rents through both channels. Without tying, if app prices could be negative, app producers would compete prices down to $p_{B2} = -r_m$. The monopolist would then again capture the consumer surplus $r_m + \Delta$ by charging $p_A =$

[17]In the Android example, the system not only gets access to consumers through its search app, but also, for instance, through a media player.

$v_A + v_{B1} + \Delta + r_m$. Combined with its advertising income through A, this would yield profits $\pi_A = v_A + v_{B1} + \Delta + 2r_m$. Again tying would be the worse outcome for the monopolist.

If prices are restricted to be non-negative however, we have $p_{B2} = 0$ which implies that the monopolist can only charge $p_A = v_A + v_{B1} + \Delta$, so its profits equal $\pi_A = v_A + v_{B1} + \Delta + r_m$, including only the advertising rents through its own system. The monopolist can no longer extract $B2$'s advertising rents through a higher p_A. As a result, tying strictly increases its profits if ad rents are sufficiently large: $r_m > \Delta$. This is the main argument in Choi and Jeon (2021).

However, the no-tying result is restored if we allow for side payments between rival app producers and monopolist (see the discussion in Choi and Jeon, 2021, section IV.F). In that case, the monopolist is better off selling system access to the entrants in the form of a transfer $t = r_m$ for each user of $B2$ that it allows on the platform. Equilibrium prices are again $p_{B2} = 0$ and $p_A = v_A + v_{B1} + \Delta$ but due to the transfer, profits of the monopolist are back at $\pi_A = v_A + v_{B1} + \Delta + 2r_m$ so it would refrain from tying.[18]

But tying is again the dominant strategy if we allow for a singlehoming premium $R > \Delta$. With tying, the monopolist would again set $p = v_A + v_{B1} + \Delta$, now earning $\pi_A = v_A + v_{B1} + 2r_m + R$. As consumers singlehome, the monopolist can extract the complete value of ad impressions from advertisers, both those impressions delivered through the app $B1$, and those impressions delivered through the system A (which, as explained above, we assume to include other apps providing different services to consumers). Without tying we again get $p_{B2} = 0$, and competing app producers are willing to pay a transfer up to $t = r_m$ to access consumers. With $p_A = v_A + v_{B1} + \Delta$, the monopolist will earn $\pi_A = v_A + v_{B1} + \Delta + 2r_m$. Since consumers now multihome on A and $B2$ and advertisers can reach them either through A or through $B2$, the monopolist and app producer can only extract their incremental profits r_m from advertisers. If $\Delta < R$, the gain from increased consumer prices would be insufficient to compensate for the loss of the singlehoming premium R for the monopolist.

[18]Following Etro and Caffarra (2017), Choi and Jeon (2021) explore a different route towards reestablishing tying, involving instead a nonpositivity constraint on the system's price.

Hence, in this case, we would see tying in equilibrium, even if we allow for side payments:

Proposition 4 *If R is sufficiently large, tying is a dominant strategy even if prices are restricted to be non-negative and we allow for side payments.*

Though we call this tying, it is fair to say that this mechanism has the flavor of the 'contracting with externalities' models to explain vertical exclusion rather than the predatory notions underlying traditional tying models. By excluding $B2$, the advertising market is monopolized, yielding R for the industry as a whole. The monopolist thus implements the industry-profit-maximizing outcome as in Bernheim and Whinston (1998).

6 Conclusion

In this paper, we analyzed the competitive strategies of platforms operating on two-sided markets. We argued that they face a strategic seesaw: the more strongly they choose to compete on the consumer side of the market, the less fierce competition will be on the advertiser side – and vice-versa. Fierce competition on the consumer side induces consumers to singlehome. But when consumers are singlehoming, platforms gain more market power over advertisers. Hence, the more important the singlehoming revenues that can be gained on the advertising side, the more strongly the platforms will choose to compete on the consumer side.

We looked at various strategies that induce consumers to singlehome: offering less differentiated products, diversifying the product portfolio a firm offers, and designing incompatible products. We also applied the mechanism to give an alternative explanation for tying.

We used advertising as our leading example, but argued that our model applies to any business strategy for which unique access to consumers' eyeballs is profitable. For example, the platform where a consumer singlehomes is able to collect more and better data on that consumer, which also puts it at an advantage over its competitor. We therefore kept the 'advertiser' side as general as possible, only positing there is some additional benefit to a platform from having consumers visit exclusively. Whether this benefit arises from a monopoly over attention, over consumer data, or in being able to directing consumers' search through product referrals is immaterial for this analysis.

To address welfare effects, we would have to be more specific about the structure of the advertising market, also addressing e.g. the nuisance to consumers that advertising causes, and the repercussions on the product markets of advertisers. Another effect that we do not take into account in our current framework is that of platform quality. More intense competition for consumers, in particular with a zero price constraint, could boost platforms' incentives to provide quality.

It is often argued that antitrust analyses of platform markets should explicitly take their two-sidedness into account. For example, charging prices below marginal cost may not be predatory but rather a matter of subsidizing one side of the market to get the other side on board. What our analysis shows is that taking two-sidedness into account not only helps us to better understand the pricing strategy of a platform, but also the competitive stance that it takes.

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A Appendix: Intermediate values of σ

In the model of the first application, choice of content, with $0 < \sigma < 1$, we may have an equilibrium as depicted in Figure 5; this requires that in equilibrium $P^* + \sigma < 1$, so consumers that have a sufficiently high valuation for both products will multihome.

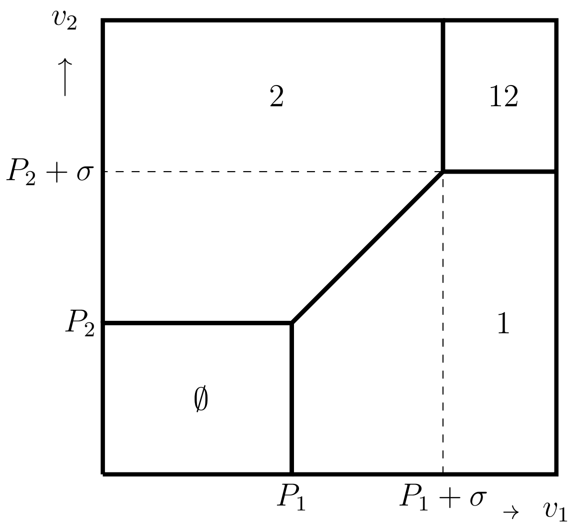

Figure 5: σ intermediate

In such an equilibrium, from Figure 5, the number of consumers that singlehomes at platform 1 is given by

$$Q_1 = \int_{P_1}^{P_1+\sigma} F(v_1 + P_2 - P_1) dF(v_1) + (1 - F(P_1 + \sigma)) F(P_2 + \sigma), \tag{10}$$

while the number of multihomers is

$$Q_{12} = (1 - F(P_1 + \sigma)) (1 - F(P_2 + \sigma)).$$

Total profits for firm 1 are

$$\pi_1 = (P_1 - C + R)Q_1 + (P_1 - C)Q_{12}.$$

Taking the first-order condition for P_1 and imposing symmetry then yields:

$$\int_{P^*}^{P^*+\sigma} F(v)dF(v) + (1 - F(P^* + \sigma)) = (P^* - C + R)\left[f(P^*)F(P^*) + \int_{P^*}^{P^*+\sigma} f(v)dF(v)\right] \\ + (P^* - C)f(P^* + \sigma)\left(1 - F(P^* + \sigma)\right).$$

If $P^* = 0$, the square of zerohomers vanishes; firms obtain profits $R-C$ from singlehoming, and $-C$ from multihoming consumers, yielding profits

$$\pi_1(P = 0) = (R - C)\frac{1}{2}\left(1 - (1 - F(\sigma))^2\right) - C\left(1 - F(\sigma)\right)^2.$$

Example Again using our uniform example with $C = 0$, the first-order condition collapses to

$$P^* = \frac{1 - \sigma(1 + R) + \frac{1}{2}\sigma^2}{2 + R - \sigma},$$

and profits follow straightforwardly.[19]

B Appendix: Proofs

Proof of proposition 1

Define the volumes of users as V_A, V_B, V_{AB}, V_0, with

$$V_A = Prob\left[u_A = \max\left\{0, u_A, u_B, u_{AB}\right\}\right] \text{ etc.}$$

The case $R = 0$: In this case, at symmetric prices P, firm A's profit is given by

$$\pi_A(P, \sigma) = (P - C)(V_A + V_{AB}).$$

For $\sigma = 0$, independent products, $u_{AB} > u_B$ as long as $u_A > 0$, so sales are made for

[19]This expression corresponds with the $\sigma = 0$ situation in that limit. We cannot use it to compare with the opposite polar situation, $\sigma = 1$, as this derivation suppose a multihoming region to exist, which as mentioned above requires $P^* + \sigma < 1$.

any consumer with $u_A > 0$, and in particular profits only depend on u_A and not on u_B. For $\sigma > 0$, the volume of consumers, at the same prices P, will be strictly lower than with $\sigma = 0$, since those with $0 < u_A < \sigma$ and $u_B > u_A$ will not be included in the volume.

Take prices P as the symmetric equilibrium price for some $\sigma > 0$. Then by the above reasoning, $\pi_A(P, \sigma = 0) > \pi_A(P, \sigma)$. Since at $\sigma = 0$, A's profits no longer depend on u_B, in the $\sigma = 0$ equilibrium prices P^* are chosen to maximize own profits. As a result,

$$\pi_A(P^*, \sigma = 0) \geq \pi_A(P, \sigma = 0) > \pi_A(P, \sigma).$$

The case R large: First, for $\sigma = 1$, no consumers consume both products AB. At $P = 0$, half of consumers consume A, the other half B, and industry profits are $R - C$, with each firm obtaining half of that. For $R > \max\{2 + C, 2\}$, $P = 0$ is the equilibrium: a deviation to $P > 0$ always reduces profits: consumer volume decreases, and $P = 1$ is the choke price, so the loss in R-income will always dominate the gain from P-income.

Next, we prove that for any $\epsilon > 0$, choosing $\sigma < 1 - \epsilon$ will lead to lower industry profits than $R - C$, for R sufficiently large. We do so by showing that maximum industry profits for such lower σ in any symmetric configuration are dominated by these $\sigma = 1$ profits.

So assume $\sigma < 1 - \epsilon$. Industry profits at symmetric prices P are given by

$$\pi = (P - C + R)(1 - V_0) + (P - C - R)V_{AB},$$

since multihomers generate extra profits $P - C$, but this comes at the expense of their gains R from advertisers. We can provide an upper bound on these profits, by focusing on two different cases: either $P \leq \frac{\epsilon}{2}$ or $P > \frac{\epsilon}{2}$

If $P \leq \frac{\epsilon}{2}$ we have that at least some consumers will multihome, V_{AB} will be positive. In particular, we can obtain the lower bound $V_{AB} = (1 - F(P + \sigma))^2 > \frac{\alpha^2 \epsilon^2}{4}$. Here, $\alpha > 0$ is the minimum of $f(v)$. Also, $V_{AB} \leq 1$.

The concern in this case might be that the extra $P - C$ gains on those multihomers (in particular if C is large negative, i.e. a lot of gains from advertising even on multihoming consumers) could outweigh the loss from R on these consumers. To see that this is ruled out if we take R sufficiently large, note that in this case we can provide an upper bound on those industry profits,

$$\pi < 2(P - C) + R - R\frac{\alpha^2\epsilon^2}{4}$$

using the $V_{AB} \leq 1$ for the first part, and the lower bound on V_{AB} on the second part.

Now choose $R > \max\{C, \frac{8}{\alpha^2\epsilon^2}, \frac{8(1-\frac{C}{2})}{\alpha^2\epsilon^2}\}$ (again, note C might be positive or negative), so that by plugging in the appropriate bounds[20]

$$\pi < \epsilon - 2C + R - 2(1 - \frac{C}{2}) = R - C + \epsilon - 2 < R - C,$$

showing industry profits are lower than industry profits at equilibrium for $\sigma = 1$.

In the other case, conversely, for $P \geq \frac{\epsilon}{2}$, it might be that no consumers multihome even though $\sigma < 1$. But in this case, we can again provide an estimate for an upper bound on industry profits, and see that this is below the $\sigma = 1$ result, as follows. Let us choose R sufficiently large again, $R > \frac{8}{\alpha^2\epsilon^2} - (1 - C)$, then we find

$$\pi \leq (P - C + R)(1 - V_0) = (P - C + R)(1 - F^2(P)) < (1 - C + R)(1 - \alpha^2 P^2)$$
$$< (1 - C + R)(1 - \alpha^2\epsilon^2/4) = 1 + R - C - (1 - C)\frac{\alpha^2\epsilon^2}{4} - R\frac{\alpha^2\epsilon^2}{4} < R - C - 1$$

where the last inequality again follows from our lower bound on R.

In conclusion, profits at any $\sigma < 1 - \epsilon$ are always dominated by equilibrium profits at $\sigma = 1$ for R sufficiently large.

Q.E.D.

Proof of proposition 2

The case $R = 0$: With diversification, we have two separate competitive duopolies, one for the A-type products, the other for the B-type products. Let us call the resulting duopoly equilibrium price P_D; in the symmetric equilibrium, all four products are sold at

[20]i.e., $2P < \epsilon$, and $R\frac{\alpha^2\epsilon^2}{4} > 2(1 - \frac{C}{2})$

the same price, and each firm sells to half of each market.

Looking instead at the specialized firms, we see these are no direct competitors, and for $R = 0$ their profits are not affected by their rival's pricing. Each firm could choose to set prices equal to P_D, leading to the exact same volumes and profits as for the diversified firms. However, firms can unilaterally adapt prices to optimize profits, and their monopoly profits strictly dominate.

The case when R is large: For the diversification case, we have two duopolies, leading to $P = 0$ pricing in each market for R large, as in proposition 1. In that equilibrium, both firms get half the A-market and half the B market. These halves being independent, each firm has 1/4 single-homing consumers, so that, with $P = 0$, $\pi_{AB} = R/4$.

On the other hand, for the specialization case, let us denote the total market covered by the AA firm as V_A, so, with symmetric prices P,

$$V_A = 1 - F^2(P).$$

Then profits for the AA' firm are

$$\pi_{AA'} = (P - C)V_A + RV_A(1 - V_B)$$

In the symmetric equilibrium, where $V_A = V_B$, we find the first-order condition for an interior solution

$$V_A = -\frac{P - C + R}{P' - R}, \quad \text{with} \quad P' = 1/\frac{dV_A}{dP} = -\frac{1}{f(P)F(P)}.$$

Since $f(P) > \alpha$ for any P, we find that as $R \to \infty$, $V_A \to 1$, so that $V_A(1 - V_B)$ will go to zero for R big. As a result, in the limit for large R, in the symmetric equilibrium $\pi_{AB} > \pi_{AA'}$.

Q.E.D.

Proof of proposition 3

The case $R = 0$ corresponds to the bundling model in Zhou (2017), who proves

(Proposition 1.i) that standardization always produces higher profits than incompatibility for two firms.

The case when R is large: we have two duopolies, leading to $P = 0$ pricing in each market for R large, as in proposition 1. In that situation, profits are $\frac{R}{2}$ for bundling versus $\frac{R}{4}$ for compatibility.

Q.E.D.