Competition with List Prices

Marco Haan[a,*], Pim Heijnen[a], Martin Obradovits[b]

[a]*Faculty of Economics and Business, University of Groningen, P.O. Box 800, 9700AV, the Netherlands*
[b]*Department of Economics, University of Innsbruck, Universitätsstraße 15, 6020 Innsbruck, Austria*

Abstract

Retail prices in stores are often lower than widely advertised list prices. We study the competitive role of such list prices in a homogeneous product duopoly where firms first set list prices before setting possibly reduced retail prices. Building on Varian (1980), we assume that some consumers observe no prices, some observe all prices, and some only observe the more salient list prices. We show that when the latter group chooses myopically, firms' ability to use list prices lowers average transaction prices. This effect is weakened when these consumers are rational. The possibility to use list prices facilitates collusion.

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1. Introduction

In many markets, in-store prices are frequently lower than the prices that are widely advertised. For example, electronics, fashion, or furniture retailers often advertise prices on television, radio, in printed catalogs, or via sponsored Internet ads, but then offer further discounts in-store and/or on their websites.[1]

*Corresponding author

Email addresses: m.a.haan@rug.nl (Marco Haan), p.heijnen@rug.nl (Pim Heijnen), martin.obradovits@uibk.ac.at (Martin Obradovits)

[1]Examples of such further discounts are daily promotions and clearance sales.

Similarly, manufacturers often quote a list price or suggested retail price in their ads or on their websites, but it is hardly a secret that the actual price consumers will have to pay is usually lower. In the Dutch retail gasoline market, majors operate numerous outlets that all charge different prices, but use a recommended retail price that is widely publicized.[2] Consumers know that they will never face a retail price that is higher than the recommended retail price of the brand they visit. Often, the price will be significantly lower. For ease of exposition, we will refer to the originally advertised/quoted prices as list prices in the remainder of this paper.[3]

Arguably, if such list prices are less transient and more visible than the actual retail prices set, some consumers may base their purchase decisions solely or primarily on them. Hence, retailers may be able to strategically use list prices to steer some consumer groups towards them, even though what ultimately matters to consumers is the actual retail price they will face. On the other hand, publicizing a low list price restricts a firm's pricing flexibility and may provoke aggressive discount competition by its rivals. This is particularly true since, by using price comparison websites, mobile phone apps, etc., in modern marketplaces there will typically also be a group of consumers that *is* well informed about the current, actual retail prices.[4] It is precisely the implications of these aspects that we explore in this paper.

In our model, two firms sell a homogeneous product and compete in prices in a two-stage game.[5] In the first stage, they set list prices. In the second stage, after having observed each other's list price, they set retail prices. We build on the seminal Varian (1980) framework, where consumers are either informed and buy from the cheapest firm, or are uninformed and pick a firm

[2]See e.g. `http://www.nu.nl/brandstof`.

[3]Indeed, Merriam-Webster defines a list price as "the basic price of an item as published in a catalog, price list, or advertisement before any discounts are taken" (see `https://www.merriam-webster.com/dictionary/list\%20price`).

[4]Coming back to the example of the Dutch retail gasoline market, this group may consist of consumers using popular mobile apps for gasoline price comparison such as "DirectLease Tankservice" and "ANWB Onderweg". Other consumers may be less well informed and just take into account the recommended retail prices publicized by the different brands (which are, next to the aforementioned website, also prominently displayed at gasoline stations), still others may just buy at a random station when they run out of fuel.

[5]While our analysis focuses on homogeneous products, our results do not hinge on such homogeneity. See the discussion in Section 3.4.

at random. We introduce a third type: *partially informed* consumers that are uninformed about retail prices, but *are* informed about list prices, simply because these are more prominent.

Crucially, we assume that list prices are an upper bound on the retail prices that can be set. There can be many reasons for this. Firms may fear reputational losses when surprising consumers with a retail price that exceeds their list price, resulting in a decrease in future sales. Consumers may outright reject such a retail price due to loss aversion, anger, or other behavioral reasons, rendering the practice unprofitable.[6] Also, many countries simply have laws that prohibit such misleading advertising.[7]

In our model, we study how the use of list prices affects product market competition. In a competitive context, does the possible use of list prices benefit firms? Do higher list prices imply higher retail prices? How frequent and how deep are the discounts granted off list prices? Are consumers better off as they (or at least some of them) become better informed, or more sophisticated? Also, do list prices facilitate collusion? Does collusion in list prices raise retail prices and, if so, how?

In our baseline analysis, we assume that partially informed consumers are myopic and simply go to the firm with the lower list price. We then have a mixed-strategy equilibrium in the list-price stage, often followed by a mixed-strategy equilibrium in the retail-price stage. It is hard to explicitly characterize the equilibrium distribution of list prices: this involves solving a functional differential equation, where the solutions in different intervals stem from interdependent differential equations. For part of the parameter space, we can provide a semi-analytic solution. For all other cases, the equilibrium can be approximated using a simple numerical method.

With myopic consumers, we find the following. Firms always use list prices that effectively constrain their retail prices. The firm with the higher list price offers more frequent and deeper discounts.[8] With list prices sufficiently close to each other, this firm will even set a lower retail price on average. On aggregate, the use of list prices decreases expected profits and

[6]See Bruttel (2018) for experimental evidence that demand tends to drop sharply for prices that exceed a recommended price, even if the latter has no informational content.

[7]See e.g. Rhodes and Wilson (2018) for a discussion of false-advertising regulations in the US and the European Union.

[8]The only exception is when firms' list prices are so far apart that the firm with the higher list price has no incentive anymore to compete for informed consumers.

increases consumer surplus. Firms face a prisoner's dilemma: each has an incentive to use list prices to try to attract the partially informed, yet when both do, expected profits are lower.

We often find search externalities: having better informed consumers leads to lower average prices for all. This is the case when uninformed consumers become either partially or fully informed. When partially informed become fully informed, the effect is however ambiguous. For a given share of uninformed consumers, firms prefer a balanced mix of fully and partially informed consumers; harsh competition for either group is unfavorable.

Solving for the case of rational consumers introduces further complexity. Note that in some subgames, the pricing equilibrium derived for the myopic case has partially informed consumers buying from the firm with the higher expected retail price. With rational consumers, the subgame equilibrium in such cases requires that partially informed consumers distribute themselves across firms such that their expected prices are equalized. Undercutting the competitor's list price thus no longer attracts all partially informed consumers, which reduces the incentive to do so. As a result, if the number of informed consumers is sufficiently large, firms no longer use effective list prices. Otherwise, we again have an equilibrium in mixed strategies.[9] Also in this case, we have to solve numerically. But this becomes more difficult as the equilibrium list-price distribution may involve multiple mass points and gaps.

Compared to the myopic case, average transactions prices are now higher. Firms thus benefit from facing rational rather than myopic consumers. Competition is less fierce in the list-price stage, which in turn relaxes it in the retail-price stage. In the terminology of Armstrong (2015), we thus have a *ripoff externality* when consumers become more strategically savvy and better understand the game being played.[10]

We also investigate how the ability to use list prices affects collusion. Successful collusion in list prices also increases average retail prices in our model. We thus provide a novel theory of harm for list-price collusion.[11] We

[9]Technically, the lack of a pure-strategy equilibrium is no longer caused by the profit function being discontinuous, but rather by it failing to be quasi-concave.

[10]Unfortunately, further comparative statics results for the rational case are difficult to obtain, as the instability of the mixed-strategy equilibria make precise numerical approximations infeasible with the available methods.

[11]In competition law, a theory of harm is a theoretical underpinning of why firm behavior

also show that list prices facilitate collusion in a world with myopic consumers and grim-trigger strategies. In a nutshell, the possibility to use list prices does not affect the perfectly collusive outcome, but does lower punishment profits. Defection profits may be higher, but this does not outweigh the lower punishment profits.

As noted, we study a two-stage game with interlinked price competition, where firms often mix in both stages. To our knowledge, the only other model with that feature is Obradovits (2014), which studies competition under a specific intertemporal price regulation. Another feature of our model is that, with rational consumers, their strategic behavior may involve mixing which firm to visit. In Janssen et al. (2005), uninformed consumers also mix, but only in whether to enter the market, not in which firm to visit. As in our model list prices serve to steer the partially uninformed consumers, our work is also connected to models of price-directed consumer search, see e.g. Haan et al. (2018), Choi et al. (2018), and, in particular, Ding and Zhang (2018).

Our paper fits a small literature on list prices that serve as an upper bound on retail prices. Myatt and Ronayne (2019) also consider a two-stage modification of Varian (1980) where firms first set binding list prices and then retail prices. They do not have partially informed consumers, and focus on asymmetric pure-strategy equilibria with stable price dispersion. In equilibrium, firms never use discounts off list prices. In Díaz et al. (2009), list prices also enable pure-strategy equilibria where these otherwise do not exist, but in the context of capacity constraints. Committing to a low list price relaxes competition in the discounting stage. The use of list prices then increases profits.

Gill and Thanassoulis (2016) study a Hotelling model with price takers (that always buy at list prices) and bargainers (that obtain an endogenously determined discount with some probability). The ability to give discounts increases profits and reduces consumer surplus. In Anderson et al. (2019), firms offer personalized discounts from posted list prices. In equilibrium, 'captive consumers' (who strongly prefer some product) buy at the list price, while 'contested consumers' receive poaching and retention offers. The discounting stage yields a mixed-strategy equilibrium, but there is a pure-strategy equilibrium in list prices. The effect on prices and profits is ambiguous.

In Rao (1991), a national brand and a private label first set list prices,

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restricts competition and thereby lowers (consumer) welfare.

then choose the depth of discounts, and finally their frequency. In Chen and Rosenthal (1996a,b), firms use a binding list price as a commitment to convince potential buyers to further inspect their product. In Banks and Moorthy (1999), firms use list and promotional prices to price discriminate between consumers with high and low search costs.

Other papers consider list or recommended retail prices that are non-binding. Some focus on vertical relations. Buehler and Gärtner (2013) argue that manufacturers are better informed about demand and use recommended prices to convey this information to retailers. In Lubensky (2017) it is consumers that are better informed about market conditions. In Harrington and Ye (2019), intermediate goods producers may collude on high list prices to signal high costs to prospective buyers, hence affecting bargaining. Boshoff et al. (2018) note that non-binding price announcements can increase collusive profits by reducing asymmetric information between firms. Other such theories are discussed in Boshoff and Paha (2021); see also Andreu et al. (2020).

Our paper is also related to the behavioral industrial organization literature, where firms try to exploit boundedly rational consumers. In Puppe and Rosenkranz (2011), firms may benefit from recommended prices if consumers are loss averse. In Heidhues and Kőszegi (2014), high list prices set for an extended period serve as a reference price. This boosts demand during sales, and hence can increase profits. Paha (2019) studies list price collusion when the willingness-to-pay of loss-averse consumers is anchored to list prices.

Lastly, our model shares characteristics with the literature on competitive couponing (Shaffer and Zhang, 1995; Bester and Petrakis, 1996), where firms set regular prices, but can additionally send out coupons that grant discounted prices. In our model, such price discrimination is not feasible.

The remainder of this paper is organized as follows. In Section 2, we introduce the model. Section 3 analyzes the game with myopic partially informed consumers. In Section 4, we explore the case where partially informed consumers are rational. Section 5 examines the effects of, and the scope for, collusion. We conclude in Section 6. All formal proofs are relegated to Appendix A. Appendix B features various aspects of our numerical analysis.

2. The game

We consider a market with two risk-neutral, profit-maximizing firms $i = 1, 2$ that sell a homogeneous good and compete in prices. Their marginal costs of production are normalized to zero. A unit mass of consumers have unit demand and a common willingness to pay that is normalized to one. The following events unfold. First, each firm simultaneously and unilaterally chooses its list price P_i. Second, after having observed all list prices, each firm decides on the retail price p_i that it charges in its store. Reflecting the discussion in the introduction, we impose that a firm's retail price cannot exceed its list price, so $p_i \leq P_i$.[12] Third, consumers make purchase decisions.

There are three types of consumers. A fraction $1 - \lambda - \mu$ is uninformed. They pick a firm at random and buy there, provided that the retail price does not exceed their willingness to pay. A fraction λ is fully informed. These consumers observe all retail prices and buy from the cheapest firm. Hence, these two consumer types correspond to the uninformed and informed consumers in the classic Varian (1980) model. But we also assume that a fraction μ of consumers is *partially informed*. These consumers only observe list prices, pick a firm based on that information and buy there, again provided that the retail price does not exceed their willingness to pay. Throughout, we assume that all consumer types have strictly positive measure, so $\lambda > 0$, $\mu > 0$ and $\lambda + \mu < 1$.

We study two scenarios. First, in Section 3, we assume that the partially informed consumers use a simple rule of thumb and go to the firm with the lowest list price. As it turns out, this is however not always the optimal thing to do: in some pricing subgames, the equilibrium then has the firm with the lower list price charging a higher retail price on average. We therefore refer to the partially informed as being myopic in this scenario. In Section 4, we modify the analysis by assuming that the partially informed are rational, and hence do not visit a firm with a higher expected retail price.

[12] Loss aversion is one reason why it may be unattractive for firms to set prices above the list price. Suppose the list price is a reference point. If the retail price exceeds the list price, then the consumer experiences a loss when purchasing the product. For sufficiently high levels of loss aversion, consumers may simply not purchase the product anymore. Such a severe reaction would make it unprofitable for firms to exceed the list price. For this argument to work, we need to assume that the uninformed consumers become aware of the list price of the firm where they intend to purchase.

3. Myopic partially informed consumers

In this section, we consider the case where partially informed consumers are myopic and buy from the firm with the lowest list price. We solve using backward induction. In Section 3.1, we characterize the equilibrium of all possible retail pricing subgames (stage 2). Then, in Section 3.2, we solve for the equilibrium in list prices (stage 1). Welfare implications are examined in Section 3.3. In Section 3.4, we briefly discuss the robustness of our results with respect to product differentiation.

3.1. Equilibrium in the pricing subgames

First, for any two list prices set in stage 1, we derive the equilibrium in stage 2. As the analysis is fairly standard, we restrict attention to the main arguments and relegate the details to Appendix A.

Preliminaries.. In case of different list prices, we refer to the firm with the lower list price as L, the other as H. Their respective list prices are denoted by P_L and P_H. Firm H will surely attract its share of uninformed consumers. Its mass of 'captive' consumers is thus given by

$$\alpha_H \equiv \frac{1-\lambda-\mu}{2}. \tag{1}$$

Firm L will also attract the μ partially informed for sure. Hence, its mass of captive consumers is

$$\alpha_L \equiv \frac{1-\lambda-\mu}{2} + \mu = \frac{1-\lambda+\mu}{2}. \tag{2}$$

The remaining $\lambda = 1 - \alpha_H - \alpha_L$ fully informed consumers buy from the cheapest firm.

Define the ratio of list prices as R, i.e.,

$$R \equiv \frac{P_H}{P_L}. \tag{3}$$

By construction, $R > 1$. In case of equal list prices, we let $R = 1$; in this case, we assume that the partially informed choose randomly which firm to visit.

Equilibrium characterization.. First, if P_L is much smaller than P_H (so R is large), firm L will simply set $p_L = P_L$, while firm H will charge $p_H = P_H$. For this to be an equilibrium, undercutting $p_L = P_L$ should not be worthwhile for H even though it attracts all informed consumers. This requires $(\alpha_H + \lambda)P_L \leq \alpha_H P_H$, so $R \geq \frac{1-\alpha_L}{\alpha_H}$. If the two list prices are closer to each other, undercutting P_L *is* profitable for H, and an equilibrium in pure strategies fails to exist.

Now suppose $P_1 = P_2 = P$, so $R = 1$. The subgame then collapses to Varian (1980) with λ informed and $1-\lambda$ uninformed consumers, and an upper bound on prices P. In equilibrium, both firms draw their price from some cumulative distribution function (CDF) $F(p)$ with support $\left[\underline{p}, P\right]$. Firm 1's expected profit from charging any $p \in \left[\underline{p}, P\right]$ is

$$\pi(p) = \left(\frac{1-\lambda}{2} + \lambda(1 - F(p))\right)p,$$

as it sells to its share $\frac{1-\lambda}{2}$ of uninformed consumers for sure, and to the mass λ informed consumers if it charges a price lower than its rival. In equilibrium, all $p \in \left[\underline{p}, P\right]$ should yield the same expected profit, $\pi(p) = \pi(P) = \frac{1-\lambda}{2}P$. Solving for $F(p)$ then gives

$$F(p) = \frac{1 + \lambda - (1-\lambda)P/p}{2\lambda},$$

with support $\left[\frac{1-\lambda}{1+\lambda}P, P\right]$.

With R sufficiently close but not equal to 1, the equilibrium is similar to that in Narasimhan (1988). That paper has (in our notation) $\alpha_L > \alpha_H$, but $P_L = P_H \equiv P$. Its equilibrium has both firms mixing on some $[\underline{p}, P]$, but in addition, firm L (with more captive consumers) has a mass point at P. The mass point assures that both firms are willing to mix on the exact same interval, which must necessarily be the case in equilibrium.[13] For R close to 1, our equilibrium is qualitatively the same: both firms mix on some $[\underline{p}, P_L]$, and in addition, firm L has a mass point at P_L.

For somewhat larger R, the above equilibrium breaks down, as H would then rather deviate to $p_H = P_H$. For such R, the equilibrium is similar to the

[13] It makes no sense for one firm to mix among prices on which the other firm puts zero probability mass.

subgame equilibrium in Obradovits (2014). The second stage in that paper has (in our notation) $P_H > P_L$, but $\alpha_L = \alpha_H$. Its equilibrium has both firms mixing on some $[\underline{p}, P_L]$, but firm L has a mass point at P_L, while firm H has one at P_H. The probability masses assure that both firms are willing to mix on the same interval $[\underline{p}, P_L]$. For intermediate R, our equilibrium is qualitatively the same.

Filling in all details, we obtain the following:

Proposition 1. *Consider list prices P_L and P_H, with $0 < P_L < P_H \leq 1$. In the equilibrium of stage 2, firm $i \in \{L, H\}$ sets its retail price equal to P_i with probability σ_i and otherwise draws it from some common distribution $F(p)$ on $[\underline{p}, P_L]$, where σ_L, σ_H, $F(p)$, $\underline{p}$, and equilibrium profits π_L and π_H are given by:*

Case *for*	A $R \leq R_0$	B $R \in (R_0, R_1)$	C $R \geq R_1$
σ_L	$\frac{\alpha_L - \alpha_H}{1-\alpha_H}$	$\frac{\alpha_H(R-1)}{1-\alpha_H-\alpha_L}$	1
σ_H	0	$\frac{(1-\alpha_H)\alpha_H R-(1-\alpha_L)\alpha_L}{(1-\alpha_L)(1-\alpha_H-\alpha_L)}$	1
$F(p)$	$\frac{1-\alpha_H-\alpha_L P_L/p}{1-\alpha_H-\alpha_L}$	$\frac{1-\alpha_L-\alpha_H P_H/p}{1-\alpha_L-\alpha_H R}$	
$\underline{p}$	$\frac{\alpha_L}{1-\alpha_H} P_L$	$\frac{\alpha_H}{1-\alpha_L} P_H$	
π_L	$\alpha_L P_L$	$\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L} P_H$	$(1-\alpha_H)P_L$
π_H	$\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L$	$\alpha_H P_H$	$\alpha_H P_H$

with $R = P_H/P_L$; $R_0 = \frac{\alpha_L(1-\alpha_L)}{\alpha_H(1-\alpha_H)}$; $R_1 = \frac{1-\alpha_L}{\alpha_H}$; $\alpha_L = \frac{1-\lambda+\mu}{2}$; $\alpha_H = \frac{1-\lambda-\mu}{2}$.

Properties of the stage 2 equilibrium.. The results we derived above already allow us to pin down some interesting implications concerning the frequency and depth of discounts that firms give vis-à-vis their list price.

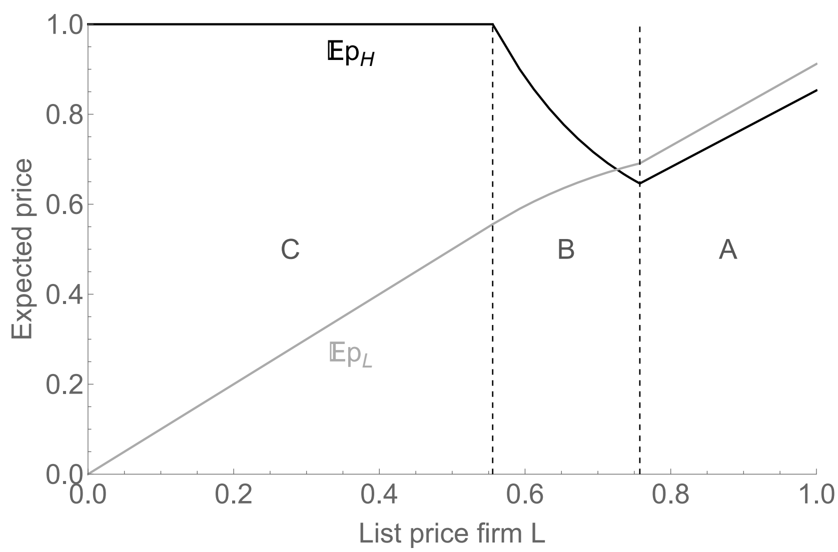
Result 1. *The minimal discount that firm H offers is $P_H - P_L$.*

When firm H uses a discount, it will always undercut the lower list price. As firm L cannot price above its list price, firm H can only possibly attract the fully informed consumers by setting a retail price lower than p_L. Offering any smaller discount would certainly be ineffective.

Result 2. *In cases A and B, firm H is more likely to offer a discount than firm L: $\sigma_H < \sigma_L$.*[14] *In case A, it always offers one.*

Intuitively, firm L has more captive consumers and hence less of an incentive to try to attract the informed. This also implies that for P_L sufficiently close to P_H, firm L charges a higher price on average. Hence, the partially informed would then be better off buying from firm H. Figure 1 illustrates this for a specific parameter combination. Here, the expected retail price of firm L exceeds that of firm H whenever $R > R^*$, with $R^* \in (R_0, R_1)$.

Figure 1: Expected retail prices as a function of P_L, with $P_H = 1$.



Expected retail price of firm L (gray line) and H (black line) as a function of P_L ($P_H = 1$, $\lambda = 0.2$, $\mu = 0.3$, dashed lines indicate boundaries between the cases in Proposition 1.

Indeed, we can show that this is always true:

Lemma 1. *There is a unique $R^* \in (R_0, R_1)$ such that the expected retail price of firm H is lower than that of firm L if and only if $R < R^*$.*

For all combinations of list prices such that $R < R^*$, the partially informed consumers thus go against their own best interest when following the

[14] In case B, note that $\sigma_H < \sigma_L$ reduces to $R < \frac{1-\alpha_L}{\alpha_H} = R_1$, which is true in that case.

simple rule of thumb of buying from the firm with the lower list price. In Section 4, we study the case when the partially informed consumers are rational and adjust their behavior accordingly. In the following subsections, we first continue the analysis for the case of myopic partially informed consumers.

3.2. Equilibrium choice of list prices

We now solve for the equilibrium of the first stage. We refer to list prices as being *effective* if they are strictly lower than the consumers' willingness to pay. Otherwise, they have no bite.

Equilibrium properties.. It is easy to see that in any candidate pure-strategy equilibrium at least one firm would be better off slightly undercutting the list price of its rival.[15] Using fairly standard arguments, we can then show the following:

Proposition 2. *Suppose the partially informed consumers are myopic. Any symmetric equilibrium then has firms sampling list prices from an atomless CDF $G(P)$ with support $[\underline{P}, 1]$, where $\underline{P} \in \left[\frac{\alpha_H}{1-\alpha_H}, \frac{1}{R_0}\right)$.*

Hence, as in Varian (1980), firms mix across list prices on some interval $[\underline{P}, 1]$, where the upper bound is given by consumers' willingness to pay. The lower bound is always such that Case B as defined in Proposition 1 can occur.[16] List prices below $\frac{\alpha_H}{1-\alpha_H}$ are dominated by setting $P_i = p_i = 1$.

We can next establish the following:

Proposition 3. *If consumers are myopic, then in equilibrium, effective list prices are always used. The possibility to use list prices strictly decreases average equilibrium prices and profits. An upper bound on profits is given by*

$$\bar{\pi} \equiv \min\left\{\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H}, \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}\right\}. \tag{4}$$

[15] From Proposition 1, firm L's equilibrium profit is weakly increasing in P_L and strictly so if $R \leq R_0$. Hence, an equilibrium with $P_L^* < P_H^*$ fails to exist, as firm L is better off setting P_L closer to P_H^*. If both firms set $P^* > 0$, each has profits $\pi^* = \frac{1-\lambda}{2}P^*$. A firm that undercuts P^* ends up as firm L in Case A of Proposition 1, which yields deviation profits arbitrarily close to $\alpha_L P^* = \frac{1-\lambda+\mu}{2}P^* > \pi^*$, so this deviation is profitable. But $P^* = 0$ cannot be an equilibrium either: deviating to a higher list price then yields positive profits.

[16] Since $\overline{P}/\underline{P} > R_0$, there is always a positive probability that $P_H/P_L > R_0$.

That list prices are used in equilibrium follows directly from the observation that firms use mixed strategies. That they decrease profits can be understood as follows. Firms compete for partially informed consumers with list prices; the lower the list price, the more likely a firm is to attract those consumers. However, list prices put a ceiling on retail prices, so their use pushes down firms' feasible pricing ranges, resulting in lower transaction prices on average. Firms would like to commit not to use list prices, yet they have a unilateral incentive to do so. Thus, this is a prisoner's dilemma.

Partially informed consumers can indeed have a stark impact on firms' equilibrium profits. Since $\alpha_H = \frac{1-\lambda-\mu}{2}$, the profit bound in (4) tends to zero as $\mu \to 1-\lambda$ so that the number of uninformed consumers goes to zero. Hence, as in Varian (1980), having uninformed consumers is necessary for firms to make positive profits.

Equilibrium characterization. Note that Proposition 3 does not pin down equilibrium profits. In case A of Proposition 1, the profits of a firm setting $P_i = 1$ depend on the list price of its rival. Hence, equilibrium profits cannot be directly determined. The different intervals in Proposition 1 yield a second complication in deriving the mixed-strategy equilibrium. Profits of a firm depend not only on whether its list price is higher or lower than its rival's, but also on which case in Proposition 1 occurs. This greatly complicates matters. To get some more grip on the equilibrium distribution of list prices, we proceed as follows. Using Proposition 1, the expected profits of a firm charging P equal

$$\Pi(P) = \overbrace{G\left(\frac{P}{R_0}\right)\alpha_H P}^{P\text{ highest, case B or C}} + \overbrace{\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}\int_{\frac{P}{R_0}}^{P} sdG(s)}^{P\text{ highest, case A}} + \overbrace{[G(PR_0)-G(P)]\alpha_L P}^{P\text{ lowest, case A}}$$
$$+ \underbrace{\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}\int_{PR_0}^{PR_1} sdG(s)}_{P\text{ lowest, case B}} + \underbrace{[1-G(PR_1)](1-\alpha_H)P}_{P\text{ lowest, case C}}. \quad (5)$$

This can be seen as follows. If firm i sets some list price P_i, firm j may set a lower P_j such that $P_i/P_j > R_0$. Given that P_j is drawn from G, the probability that this happens is $G(P_i/R_0)$. If it does, we are in case B or C in Proposition 1, and firm i has profits $\alpha_H P_i$. This yields the first term in (5). Second, for any $P_j \in (P_i/R_0, P_i)$, we end up in case A with i having the higher list price, so its profits are $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}P_j$. Integrating over all relevant P_j gives the second term. The remaining terms follow in a similar fashion.

For an equilibrium, we need that the right-hand side of (5) is constant for all $P \in [\underline{P}, 1]$. As we show in detail in Appendix A, this implies the following equilibrium structure for (a relatively large) part of the parameter space:

Proposition 4. *Suppose that the partially informed consumers are myopic. For a subset of the parameter space, the symmetric equilibrium distribution of list prices then takes the form*

$$G(P) = \begin{cases} a + b_{1,1}P^{\kappa_1} + b_{1,2}P^{\kappa_2} & \text{for } P \in \left[\underline{P}, \frac{1}{R_0}\right) \\ 1 - b_2 P^{\frac{\kappa_1+\kappa_2}{2}} & \text{for } P \in \left[\frac{1}{R_0}, \underline{P}R_0\right) \\ a + b_{3,1}P^{\kappa_1} + b_{3,2}P^{\kappa_2} & \text{for } P \in [\underline{P}R_0, 1], \end{cases} \tag{6}$$

where a, b's, κ's, and $\underline{P}$ are all functions of the parameters λ and μ (details in the proof). For this solution to apply, it is sufficient that $\lambda \geq 0.385$.

For parameter values not covered by Proposition 4, we use a numerical approximation to find $G(P)$ on a discretized action space. Details about this procedure can be found in Appendix B.1.

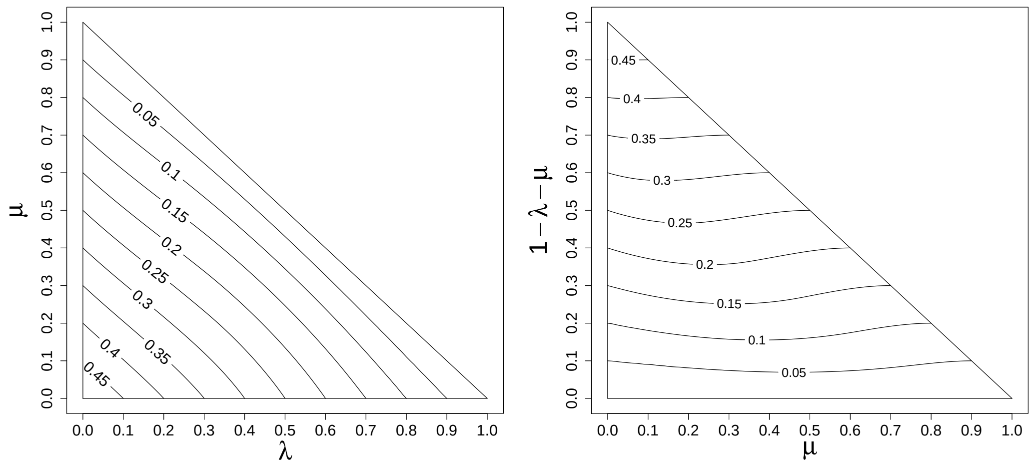
3.3. Welfare effects

Above we have characterized the equilibrium with myopic consumers. For some parameter values, we may obtain the numerical solution implied by Proposition 4. For other parameter values, we have to do a numerical approximation. In this section, we use those results to analyze welfare effects. We focus on the comparative statics effects on profits; as all consumers buy, total welfare always equals 1 so the effects on consumer welfare are simply the opposite.

Figure 2(a) shows a contour plot of the equilibrium profits in (λ, μ)-space. Moving up in this graph thus implies keeping the number of informed (λ) fixed, while increasing the number of partially informed (μ) at the expense of the number of uninformed ($1 - \lambda - \mu$). Similarly, moving to the right implies shifting consumers from uninformed to informed.

From the figure, profits are strictly decreasing in λ and μ, tending to zero as $\lambda + \mu \to 1$ (cf. the second paragraph after Proposition 3). Thus, when the share of uninformed consumers in the market decreases, firms are

Figure 2: Contour plot of equilibrium profits.



(a) In (λ, μ)-space.

(b) In $(\mu, 1-\lambda-\mu)$-space.

For values $\lambda \in \{0, 0.01, \ldots, 0.98\}$, $\mu \in \{0, 0.01, \ldots, 0.98\}$, $\lambda + \mu \leq 0.98$.

unambiguously worse off, no matter whether this is because the proportion of fully informed or partially informed increases.

Figure 2(b) gives the same information as Figure 2(a), but now in $(\mu, 1 - \lambda - \mu)$-space. Moving down in the graph means that uninformed consumers become fully informed. As just observed, this decreases profits. Moving to the left means that partially informed consumers become fully informed. From the graph, the effect on firm profits is non-monotonic. If the number of partially informed is low, fully informing more of them decreases profits. But if their number is high, doing so increases profits.

Note that with either $\lambda = 0$ or $\mu = 0$, we are back to Varian (1980) competition: if $\mu = 0$, competition is at the retail level; with $\lambda = 0$, it is at the list price level. If λ, $\mu > 0$, there is competition at both levels. This benefits firms relative to the case of fierce competition at either level.[17]

[17]In Varian (1980), equilibrium profits are determined by the share of uninformed consumers. In our model, for a fixed share of uninformed consumers (moving on a horizontal line in Figure 2(b)), profits never fall short of those with $\mu = 0$ (on the far left of the graph) or $\lambda = 0$ (on the far right).

Summing up, we find the following:

Numerical Result 1. *Suppose the partially informed consumers are myopic. When uninformed consumers become either partially or fully informed, profits decrease. When partially informed consumers become fully informed, profits decrease when their share is relatively low, while profits increase when their share is relatively high.*

Armstrong (2015) gives a general analysis of models with both informed and less informed consumers ("savvy" and "non-savvy" in his more general terminology). In his analysis, there is a *search externality* when each type of consumer is better off when the number of savvy consumers increases. There is a *ripoff externality* if the opposite is true.

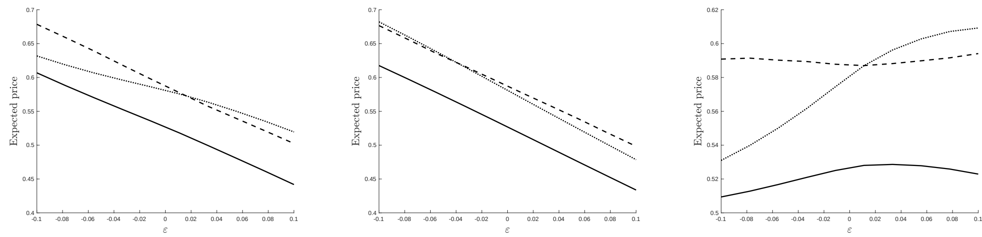
Our model not only has "savvy" and "non-savvy" consumers, but also "partly savvy" ones. It is interesting to see how an increase in savviness affects these consumer types individually. We do so for one particular parameter configuration in Figure 3.[18] The panels show the effect of fully informing uninformed consumers (a), partially informing uninformed consumers (b), and fully informing partially informed consumers (c). The dashed lines give the average price paid by the uninformed, the gray solid lines that paid by the partially informed, the black solid lines that paid by the fully informed.

From the graph, informing uninformed consumers (either partially or fully) yields a search externality: due to such a change, the average price paid by all types of consumers decreases. Hence, the lower profits (and hence higher consumer surplus) we found in Figure 2 benefit all consumers. But the effect of further informing partially informed consumers is ambiguous for each consumer type. We already saw that for the aggregate effect in Figure 2(b).

From Figure 3, partially informed consumers may be worse off than uninformed consumers. The uninformed just pick a firm at random, while the partially informed choose the firm with the lower list price, which might charge a higher actual retail price on average.

[18]For other parameter configurations, the graphs look qualitatively similar. Contour plots of the expected prices paid by the different consumer groups as functions of λ and μ (similar to Figure 2) are available from the authors upon request.

Figure 3: The effects of increasing consumer savviness.



(a) Uninformed to full. (b) Uninformed to partial. (c) Partial to full.

Average price paid by the uninformed (dashed lines), partially informed (gray solid lines) and informed (black solid lines) for varying λ and μ. Starting from the benchmark $\lambda = 0.25, \mu = 0.2$, the panels show the effect of (a) an increase in the fraction of fully informed by ε while decreasing the fraction of uninformed by ε; (b) an increase in the fraction of partially informed by ε while decreasing the fraction of uninformed by ε; (c) an increase in the fraction of fully informed by ε while decreasing the fraction of partially informed by ε.

3.4. Product Differentiation

So far, we have focused our analysis on homogeneous products. This raises the question as to whether our model mechanism may also give rise to effective (binding) list prices when products are differentiated.[19] This is indeed the case if a firm can still capture sufficiently many partially informed consumers by setting a list price slightly below its competitor's. Then, an outcome without effective list prices (so $P^* = 1$) is not an equilibrium as firms would find it profitable to undercut. For example, this may be the case if each consumer has a preference for one of the firms and only receives utility $1 - \Delta$ when consuming their less preferred product (see, e.g., Shilony (1977)). For low enough Δ, this still induces an equilibrium where effective list prices are used.[20]

[19]We thank an anonymous referee for encouraging this discussion.

[20]For illustration, suppose that this preference structure applies just to uninformed and partially informed consumers. Moreover, assume for simplicity that the partially informed believe that retail prices are equal to list prices. As fully informed consumers lack brand loyalty, we can then use all our subgame results of Subsection 3.1, the only difference being that a firm now needs to undercut the list price of its rival by at least Δ to attract the $\mu/2$ partially informed consumers that prefer its rival. For $\Delta \leq 1 - 1/R_0$, a deviation from $P^* = 1$ would induce case A of Proposition 1, for a deviation profit of $\pi_L = \alpha_L(1 - \Delta)$. With $P^* = 1$ we have $\pi = \frac{1-\lambda}{2}$, so the deviation pays if $\Delta < \frac{\mu}{1-\lambda+\mu}$. Hence, for sufficiently small product differentiation Δ, effective list prices will be used in equilibrium. It is also straightforward to show that a pure-strategy equilibrium (in the choice of list prices) fails

4. Rational partially informed consumers

Above we studied the case where partially informed consumers buy from the firm with the lowest list price. Yet, from Lemma 1, this implies that they may buy from the firm with the *highest* expected retail price. Clearly, rational consumers should not behave in such a manner. In this section, we therefore study the case of rational consumers.

Suppose that indeed $\mathbb{E}p_L > \mathbb{E}p_H$ when all partially informed buy at firm L. Some partially informed should then switch to firm H. By doing so, L gets fewer captive consumers, while H gets more. As a result, the expected retail price of firm L decreases, and that of H increases. This process continues up to the point where $\mathbb{E}p_L = \mathbb{E}p_H$.[21] For a subgame equilibrium with rational consumers, we thus need:

Definition 1. *Given list prices* (P_L, P_H), *an equilibrium of the retail pricing subgame with rational consumers consists of (possibly degenerate) CDFs* $F_L(p_L|P_L, P_H, \theta)$ *and* $F_H(p_H|P_L, P_H, \theta)$, *and a fraction* θ *of partially informed consumers that buys from firm* L, *such that*

1. *drawing* p_L *from* F_L *maximizes* L*'s profits given* $p_H \sim F_H$ *and given* θ;
2. *drawing* p_H *from* F_H *maximizes* H*'s profits given* $p_L \sim F_L$ *and given* θ;
3. *either one of the following conditions holds:*
   (a) $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ *and* $\theta = 1$;
   (b) $\mathbb{E}p_L(\theta) = \mathbb{E}p_H(\theta)$.

We proceed as follows. Section 4.1 discusses equilibrium in the retail pricing subgames. In Section 4.2, we examine the choice of list prices. Welfare implications and a comparison to the myopic case are given in Section 4.3.

to exist in that case.

[21] The only alternative would be if all partially informed buy from H and $\mathbb{E}p_L > \mathbb{E}p_H$, but that cannot be part of an equilibrium either: firm L would then have a lower list price *and* fewer loyal consumers, rendering its pricing more aggressive than its rival's such that $\mathbb{E}p_L < \mathbb{E}p_H$ (see also the proof of Lemma A.9 in Appendix A.

4.1. Adjusted pricing subgames

In the myopic case, the shares of captive consumers are given by (1) and (2). As only a fraction θ of partially informed now visit firm L, that changes to

$$\begin{aligned} \alpha_H(\theta) &= \frac{1-\lambda-\mu}{2} + (1-\theta)\mu, \\ \alpha_L(\theta) &= \frac{1-\lambda-\mu}{2} + \theta\mu. \end{aligned} \tag{7}$$

For any θ, we can directly use Proposition 1 to find the corresponding mixed-strategy equilibrium for these adapted values of α_H and α_L.

To find the equilibrium with rational consumers, we thus proceed as follows:

1. If $R \geq R^* \in (R_0, R_1)$, we have $\mathbb{E}p_L(1) \leq \mathbb{E}p_H(1)$, so the equilibrium characterization in Proposition 1 still applies.

2. If $R < R^*$, we have $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$. In that case, we have to find the value $\tilde{\theta}$ for which $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$.

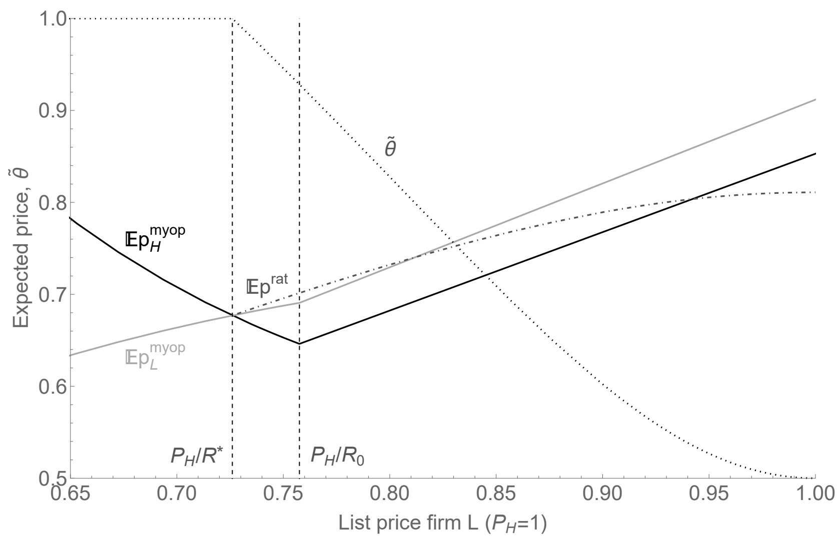
For this procedure to work, we need that such a $\tilde{\theta}$ always exists and is unique. This is indeed the case:

Lemma 2. *For any (P_L, P_H) with $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$, there is a unique $\tilde{\theta} \in \left(\frac{1}{2}, 1\right)$ such that $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$.*

Example.. As an illustration, we revisit the example in Figure 1. Figure 4 zooms in on the interval [0.65, 1] and adds the dashed-dotted line, which gives the (identical) expected price of both firms when consumers are rational and $R \leq R^*$. The dotted line gives the equilibrium share $\tilde{\theta}$ of partially informed consumers that visit firm L in this case. This decreases from 1 (if $R = R^*$) to 1/2 (if $R = 1$).

Interestingly, with rational rather than myopic consumers, expected prices are higher in some pricing subgames (when R is close below R^*) but lower in others (when R is close to 1). With rational consumers, firms become more symmetric in their share of captive consumers. If their list prices are close to each other, this leveled playing field implies more aggressive competition (cf. Narasimhan, 1988, p. 441, point 1.iii). However, if the difference in list prices is large, the playing field is already very tilted to start with. Having

Figure 4: Expected prices with myopic and rational consumers.



Expected price of firm L and firm H as a function of the list price of firm L with myopic (gray and black solid line, respectively) and with rational partially informed consumers (dashed-dotted line). Also depicted: equilibrium value $\tilde{\theta}$ (dotted line). The parameters used are $P_H = 1$, $\lambda = 0.2$, $\mu = 0.3$.

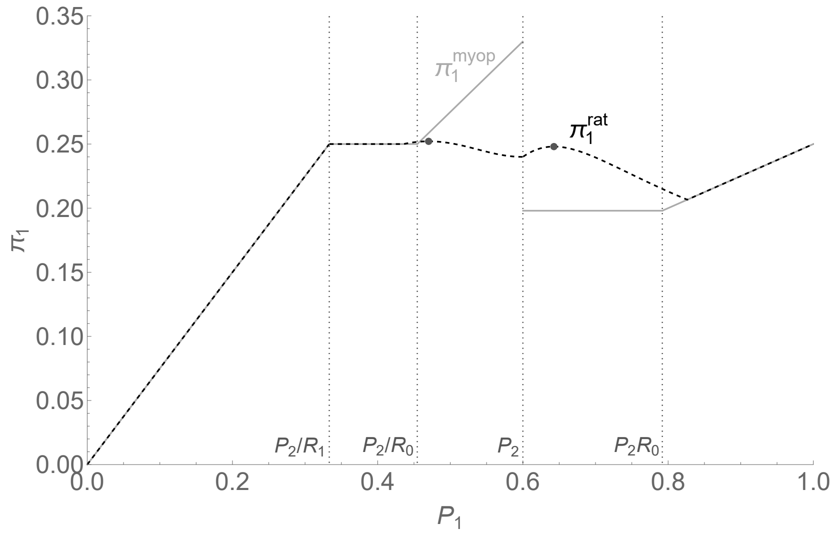
more captive consumers now only makes H more reluctant to compete for the informed, as that requires sacrificing its relatively high margin on an increased base of captive consumers. With H competing less aggressively, L follows suit.

4.2. Equilibrium choice of list prices

We next study how incentives in the first stage of the game are affected. To illustrate, Figure 5 shows the expected profits of firm 1 as a function of P_1 if $P_2 = 0.6$, again for $\lambda = 0.2$ and $\mu = 0.3$. The gray line represents the case of myopic, the black dashed line that of rational consumers.

In the myopic case, slightly undercutting P_2 attracts *all* partially informed consumers and hence implies a discrete upward jump in profits. In the rational case, it only slightly increases the number of partially informed consumers firm 1 attracts. In the figure, the best reply of firm 1 is then to choose P_1 so as to attain the local maximum between P_2/R^* and P_2 (marked by the left black circle; P_2/R^* is not shown to avoid visual clutter). But for slightly different parameter values, the best reply may instead be to choose

Figure 5: Profits of firm 1 with myopic and rational consumers.



Profits of firm 1 as a function of P_1, given $P_2 = 0.6$ ($\lambda = 0.2, \mu = 0.3$). Gray line: myopic partially informed. Black dashed line: rational partially informed.

P_1 to attain the local maximum between P_2 and P_2R^* (marked by the right black circle; again, P_2R^* is not shown) or to set $P_1 = 1$. Small changes in parameters may thus imply big shifts in the best reply of firm 1. This makes the analysis even more involved.

As can be seen in Figure 5, firm profits are now continuous, but they fail to be quasi-concave. As a result, existence of a symmetric pure-strategy equilibrium is not guaranteed. Indeed, we can show the following:

Proposition 5. *Suppose that the partially informed consumers are rational.*

- *If $\lambda \geq 1/3$, there is a unique symmetric pure-strategy equilibrium in which both firms set $P = 1$. Hence, no effective list prices are used.*
- *If $\lambda < 1/3$, a symmetric pure-strategy equilibrium fails to exist, and effective list prices are used in equilibrium.*

Starting from $P_1 = P_2 = 1$, lowering one's list price has two effects. First, it increases one's share of partially informed consumers, which increases profits. But it also makes competition for informed consumers more aggressive, which tends to decrease profits. If the number of informed consumers is sufficiently large, the second effect dominates, leaving $P = 1$ as an equilibrium. An equilibrium with $P_1 = P_2 < 1$ fails to exist: firms would then prefer to set a slightly *higher* list price than their competitor.

The equilibrium for $\lambda < 1/3$.. In this case, it is very hard to characterize the equilibrium choice of list prices. As always, a mixed-strategy equilibrium requires that each firm is indifferent between all list prices in its support. But, if list prices are sufficiently close to each other, we also need that the shares of loyal consumers adapt such that firms' expected retail prices are equalized. In turn, these endogenously determined shares affect the subgame equilibrium profits. Moreover, these profits fail to be quasi-concave.

We therefore have to resort to a numerical approximation of the equilibrium mixed-strategy choice of list prices. Since we cannot rule out that the equilibrium distribution has mass points and gaps, we cannot use the method described in Appendix B.1. Instead, we use a numerical procedure based on Mangasarian and Stone (1964). Roughly, for each parameter combination, we discretize the action space, construct the respective payoff matrix, and numerically solve a quadratic programming problem.[22] A few examples of the approximated equilibrium list price CDFs are shown in Appendix B.2. As it turns out, mass points and gaps now do indeed occur in equilibrium.

4.3. Welfare effects

We next consider the welfare implications of the possibility of using list prices when consumers are rational. From Proposition 5, if $\lambda \geq 1/3$, firms set $P = 1$ so the model coincides with the Varian (1980) benchmark. For $\lambda < 1/3$, we find a result equivalent to Proposition 3:

Proposition 6. *If the partially informed consumers are rational, then the possibility to use list prices has no effect on equilibrium profits if $\lambda \geq 1/3$, but strictly decreases average equilibrium prices and profits if $\lambda < 1/3$.*

With myopic consumers, we have from Proposition 3 that the possibility to use list prices always yields lower profits. For $\lambda \geq 1/3$, we thus immediately have that profits are higher and consumer surplus is lower when

[22]Further details can be found in Heijnen (2020). The corresponding Matlab code is available upon request. We have also confirmed our results using an alternative, evolutionary algorithm.

consumers are rational rather than myopic. For $\lambda < 1/3$, an analytical comparison is not feasible and we again have to resort to our numerical analysis.[23] We obtain the following result (see Appendix B.3 for details):

Numerical Result 2. *With rational consumers, profits are strictly higher and consumer surplus is strictly lower than with myopic consumers.*

Armstrong (2015) makes a distinction between consumers that are "savvy" since they are well-informed, and those that are *strategically savvy* in the sense that they have a good understanding of the game being played. Hence, our partially informed consumers are strategically naive if myopic, and strategically savvy if rational. Our analysis then implies a ripoff externality in this dimension: when the partially informed become strategically savvy, the consumers end up paying a higher price on average.[24] Hence, consumers as a whole would be better off if they could commit as a group to use the simple rule of thumb.

As noted, the type of equilibrium we end up in (and hence, the equilibrium profit) is highly sensitive to parameter values. This also implies that we cannot conduct accurate comparative statics, as we did in Figure 2. Neither is it possible in this context to study the effect of an increase in consumer savviness.

5. List prices and the scope for collusion

In a number of cases, antitrust authorities have been concerned about collusion in list prices, and how that could affect transaction prices. A notable example is the truck cartel in the EU, where six producers of trucks agreed

[23] Why the comparison is so hard to do analytically can also be understood from Figure 4: for some combinations of list prices, expected prices are higher in the rational case, while for others, they are lower. The net effect then depends on how often certain combinations are chosen in equilibrium.

[24] In the main text we consider cases where partially informed consumers either are all myopic, or all rational. It is straightforward to allow for a fraction $\kappa \in (0,1)$ that are rational. Note that with $\kappa = 1$, an equilibrium fraction $\tilde{\theta}$ visits firm L, while with $\kappa = 0$ we impose $\theta = 1$. If $1 - \kappa \leq \tilde{\theta}$, we have the same solution as with $\kappa = 1$: having $\kappa \geq 1 - \tilde{\theta}$ rational partially informed consumers is enough to reach the fully rational outcome. If $\kappa < 1 - \tilde{\theta}$, we get the solution described in Proposition 1, but with $\tilde{\alpha}_L = \frac{1-\lambda-\mu}{2} + (1-\kappa)\mu$ and $\tilde{\alpha}_H = \frac{1-\lambda-\mu}{2} + \kappa\mu$.

(amongst others) upon harmonizing list prices between 1997 and 2011,[25] and were fined a total of 3.7 billion euro – still the highest EU cartel fine to date. For a detailed discussion of many relevant cartel cases in Europe and the US, see Boshoff and Paha (2021). Cartelists have argued that list price collusion is really harmless, as higher list prices will simply be offset by higher rebates, leaving transaction prices unaffected. Antitrust authorities often argue otherwise, but tend to be vague concerning the theory of harm.[26] Our model may provide exactly that.

Most theoretical contributions on list-price collusion consider list prices as a starting point that serves as the basis for price negotiations between the producer and its customers (see, e.g., Harrington and Ye (2019) or Gill and Thanassoulis (2016)). Our mechanism is completely different. In our model, consumers differ in the amount of information they have, and list prices are used to try to attract partially informed consumers, while still retaining downward pricing flexibility to compete for fully informed consumers.

In this section, we thus study collusion with list prices. First, we discuss whether in our model, successful collusion in list prices indeed leads to higher transaction prices, as often argued by antitrust authorities. Second, we study whether the possibility to use list prices in itself facilitates collusion in a world where collusion is feasible in both the list price as well as the retail price stage. Third, we study how firms' ability to collude, either fully or only on list prices, is affected by consumer information.

First of all, from Propositions 3 and 6, we immediately have:

Proposition 7. *Collusion in list prices leads to higher retail prices on average.*

With perfect collusion on list prices, firms set them equal to 1. From Propositions 3 and 6, this increases average prices compared to the case that effective list prices are used.[27]

Next, we consider whether the ability to use list prices facilitates optimal collusion. In the remainder of this section, we consider an infinitely repeated version of the baseline model of Section 2, where firms are infinitely lived

[25] See https://ec.europa.eu/competition/antitrust/cases/dec_docs/39824/39824_8750_4.pdf and https://ec.europa.eu/competition/antitrust/cases/dec_docs/39824/39824_8754_5.pdf.

[26] Again, see Boshoff and Paha (2021).

[27] Recall that with rational partially informed consumers, this holds when $\lambda < 1/3$.

but a new cohort of consumers arrives each period. Firms use a common discount factor $\delta \in (0,1)$. We only consider the more tractable case of myopic consumers and restrict attention to collusion via grim trigger strategies. For such collusion to be sustainable, we need that $\delta \geq \overline{\delta} \equiv \frac{\pi^D - \pi^C}{\pi^D - \pi^N}$, with π^N denoting Nash profits, π^C collusive profits, and π^D optimal defection profits in the stage game.

Without the possibility of list prices, we are back to Varian (1980), and $\pi^N = \frac{1-\lambda}{2}$. The cartel price is $p_i = 1$, so $\pi^C = 1/2$ and $\pi^D = \frac{1+\lambda}{2}$. Hence, our benchmark critical discount factor is $\overline{\delta}_{bench} = 1/2$.

If firms can use list prices and consumers are myopic, we can use the method in Appendix B.1 to numerically find the stage-game Nash profit π^N of the full game. Perfect collusion requires $P_i = p_i = 1$. This again implies $\pi^C = 1/2$.

For defection profits, note that firms can either defect in the list-price stage or in the retail-price stage. When they do in the list-price stage, we assume that reversion to the Nash equilibrium already takes place in the retail-pricing stage of the same period. From Proposition 1, the best defection then is to marginally undercut $P_i = 1$, which yields $\pi^D = \frac{1-\lambda+\mu}{2}$. In the retail-pricing stage, the best defection is to marginally undercut $p_i = 1$, which yields $\pi^D = \frac{1+\lambda}{2}$. Firms thus prefer to defect in the list-price stage if $\mu > 2\lambda$.

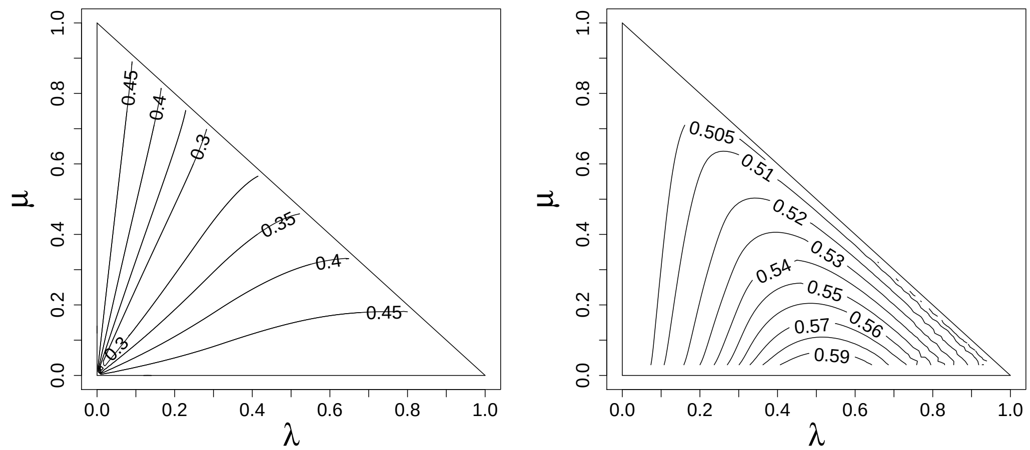
Figure 6(a) shows a contour plot of the resulting critical discount factor $\overline{\delta}$ in (λ, μ)-space. For all combinations of λ and μ, it lies below that of the benchmark case (i.e, 1/2), implying that the ability to use list prices facilitates collusion. We can indeed prove this formally:

Proposition 8. *When the partially informed consumers are myopic, the possibility to use list prices facilitates collusion.*

This can be seen as follows. First, the ability to use list prices does not affect collusive profits. Second, from Proposition 3, it lowers Nash profits, making the loss when defecting from a collusive agreement more severe. Defection profits are either unaffected by the use of list prices (if the share of informed consumers is relatively high), or they increase only moderately as a defection in the list price stage immediately provokes aggressive (Nash) pricing in the subsequent pricing stage. The more severe punishment dominates, implying that collusion is facilitated.

We now turn to the last question: how is the scope for collusion affected by consumer information? First, if firms can collude in both stages of the

Figure 6: Contour plot of critical discount factors to support collusion.



(a) Full collusion.

(b) List-price collusion.

For values $\lambda \in \{0, 0.01, \ldots, 0.98\}$, $\mu \in \{0, 0.01, \ldots, 0.98\}$, $\lambda + \mu \leq 0.98$.

game, it is apparent from the contour plot in Figure 6(a) that the effect of increasing consumer information[28]) is ambiguous. Next consider a scenario where firms can *only* collude on list prices. Figure 6(b) shows a contour plot of the critical discount factor $\overline{\delta}$ in (λ, μ)-space in this scenario. We then find:

Numerical Result 3. *When consumers are myopic and firms can only collude on list prices, collusion is facilitated when the share of partially informed consumers increases at the expense of uninformed consumers.*

Defection profits increase in the share of partially informed consumers, but the punishment also becomes more severe, as list price competition then becomes more intense. The latter effect dominates, hence the result.

6. Conclusion

In this paper, we have studied a simple homogeneous goods duopoly in the spirit of Varian (1980) where firms first set list before setting possibly

[28]i.e. partially or fully informing uninformed consumers, or fully informing partially informed consumers – which corresponds to moving to the south-east in the figure.

discounted retail prices. Next to the informed and uninformed consumers in Varian (1980), we also introduced *partially informed consumers* whose purchase decision is solely influenced by list prices. The main insights from our analysis are as follows.

First, for given list prices, whenever price discounts are granted, the firm with the higher list price gives deeper and more frequent discounts. This is because a successful discount must at least beat the other firm's list price. Moreover, the firm with the higher list price has a smaller share of captive consumers, which makes attracting informed customers relatively more lucrative.

Second, if the partially informed consumers simply buy from the firm with the lower list price, a pure-strategy equilibrium in list prices fails to exist. This is because by slightly undercutting the competitor's list price, a firm could then capture the entire mass of partially informed consumers. However, for list prices that are relatively close, the firm with the lower list price would then also have a higher retail price on average. Rational partially informed consumers understand this, and hence do not simply go to the firm with the lower list price. But this implies that list price competition for rational consumers is less fierce, exactly because slightly undercutting the competitor no longer captures all partially informed consumers.

Third, firms would be better off *not* setting list prices, as their use leads to lower transaction prices on average. This is a prisoner's dilemma, as each firm has an incentive to attract the partially informed by setting a lower list price. This is particularly true in the case of myopic consumers, and less so with rational consumers, for the reason set out above. It also implies that successful collusion on list prices leads to higher retail prices. Using list prices facilitates collusion, as competition is fiercer when a cartel breaks down.

Fourth, in the myopic case, having more informed consumers tends to lower average prices. But this is only true if uninformed consumers become better informed, making the market more competitive. If partially informed consumers become fully informed, the result may be different. Expected prices are lowest if there is fierce competition at either the list price stage (so the number of partially informed is high) or at the retail price stage (so their number is low). For intermediate values, competition is not too fierce at either level, and expected retail prices are higher as a result.

A considerable limitation of our model is its lack of tractability. In general, a closed-form solution for the list-price equilibrium cannot be obtained. In the myopic case, we can pin down the equilibrium explicitly (up to the lower bound of its support) for parts of the parameter space, and our equilibrium characterization results at least permit the use of a simple and robust numerical procedure to compute the mixed-strategy equilibrium. In the rational case, list prices will often not be used, but if they are, few characterization results are available. Our numerical results indicate highly irregular and parameter-sensitive equilibrium behavior in this case. This reduces the accuracy of our numerical results, limiting the level of detail of the analyses we can conduct.

There are several potential directions for future research. First, it would be interesting to endogenize consumer group sizes by explicitly modeling advertising decisions that inform consumers, either partially or fully. Second, search costs may be explicitly introduced into the model, hence endogenizing the behavior of different types of consumers. Third, studying the consequences of product differentiation in a more systematic manner may be informative, though this is likely to complicate the analysis even further.

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Appendix A Technical proofs

Proof of Proposition 1. Case C has been established in the main text, with profits following trivially. We prove Cases A and B using the following lemmas.

Lemma A.1. *If $R \leq R_0$, the pricing subgame has the following unique mixed-strategy equilibrium. Firm H draws its price from the CDF*

$$F_H(p) = 1 - \frac{\alpha_L \left(\frac{P_L}{p} - 1\right)}{1 - \alpha_L - \alpha_H}$$

with support $\left[\frac{\alpha_L}{1-\alpha_H} P_L, P_L\right)$. Firm L sets $p_L = P_L$ with probability

$$\sigma_L = \frac{\alpha_L - \alpha_H}{1 - \alpha_H},$$

and draws its price from $F_H(p)$ with probability $1 - \sigma_L$. Expected profits are

$$\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L \qquad \text{and} \qquad \pi_L = \alpha_L P_L.$$

Proof of Lemma A.1. We only prove that this is an equilibrium. Uniqueness can be established with the usual arguments, available upon request.

Suppose firm L plays according to the Lemma. If firm H sets $p_H \in \left[\frac{\alpha_L}{\alpha_L+\lambda} P_L, P_L\right)$, it has attracts the informed with probability $\sigma_L + (1-\sigma_L)(1 - F_L(p_H))$ yielding profits

$$p_H \left[\alpha_H + \lambda(\sigma_L + (1-\sigma_L)(1 - F_L(p_H)))\right] = \frac{(\alpha_H + \lambda)\alpha_L}{\alpha_L + \lambda} P_L.$$

Setting $p_H < \frac{\alpha_L}{\alpha_L+\lambda} P_L$ makes no sense, as it already attracts all informed. Charging $p_H = P_L$ makes no sense either: this is a mass point for L so undercutting it increases profits. As any $p_H > P_L$ will not attract the informed, the best such deviation is $p_H = P_H$. This yields $\alpha_H P_H$, which does not exceed π_H as $R \leq R_0$. Hence, H has no profitable deviation.

Suppose firm H plays according to the Lemma. If L sets $p_L \in \left[\frac{\alpha_L}{\alpha_L+\lambda} P_L, P_L\right]$, it attracts the informed with probability $1 - F_H(p_L)$ yielding profits

$$\pi_L(p_L; P_L, P_H) = p_L \left[\alpha_L + \lambda(1 - F_H(p_L))\right] = \alpha_L P_L.$$

Setting $p_L < \frac{\alpha_L}{\alpha_L+\lambda}P_L$ makes no sense, as this already attracts all informed for sure. Firm L cannot price above P_L. Hence it has no profitable deviation.

Lastly, all equilibrium objects are well-behaved, since clearly $\sigma_L \in (0,1)$, while $F_i(\frac{\alpha_L}{\alpha_L+\lambda}P_L) = 0$, $F_i(P_L) = 1$, and $\frac{dF_i(p)}{dp} = \frac{\alpha_L P_L}{p^2\lambda} > 0$. □

Lemma A.2. *If $R \in (R_0, R_1)$, the pricing subgame has the following unique mixed-strategy equilibrium. Firm H sets $p_H = P_H$ with probability*

$$\sigma_H = \frac{(1-\alpha_H)\alpha_H R}{(1-\alpha_H-\alpha_L)(1-\alpha_L)} - \frac{\alpha_L}{1-\alpha_H-\alpha_L},$$

and with probability $1-\sigma_H$ draws its price from the CDF

$$F_H(p) = \frac{1-\alpha_L-\alpha_H\left(\frac{P_H}{p}\right)}{1-\alpha_L-\alpha_H R}$$

with support $\left[\frac{\alpha_H}{1-\alpha_L}P_H, P_L\right)$. Firm L sets $p_L = P_L$ with probability

$$\sigma_L = \frac{\alpha_H(R-1)}{1-\alpha_H-\alpha_L},$$

and draws its price from $F_H(p)$ with probability $1-\sigma_L$. Expected profits are

$$\pi_H = \alpha_H P_H \qquad \text{and} \qquad \pi_L = \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P_H.$$

Proof of Lemma A.2. Again, we only prove that this is an equilibrium. Details concerning uniqueness are available upon request. Suppose L plays according to the lemma. If H sets $p_H \in \left[\frac{\alpha_H}{\alpha_H+\lambda}P_L, P_L\right)$, it has expected profits

$$p_H\left[\alpha_H + \lambda(\sigma_L + (1-\sigma_L)(1-F_L(p_H)))\right] = \alpha_H P_H.$$

If it sets $p_H = P_H$, its profits are also $\alpha_H P_H$. Setting $p_H < \frac{\alpha_H}{\alpha_H+\lambda}P_L$ makes no sense, as it already attracts all informed for sure. Setting $p_H = P_L$ makes no sense either as this is a mass point for L so undercutting it increases profits. As any $p_H > P_L$ will not attract the informed, any price in (P_L, P_H) yields lower profits than $p_H = P_H$. Hence, firm H has no profitable deviation.

Suppose firm H plays according to the lemma. If L sets $p_L \in \left[\frac{\alpha_H}{\alpha_H+\lambda}P_L, P_L\right]$, it has expected profits

$$p_L\left[\alpha_L + \lambda(\sigma_H + (1-\sigma_H)(1-F_H(p_L)))\right] = \frac{(\alpha_L+\lambda)\alpha_H}{\alpha_H+\lambda}P_H.$$

Setting $p_L < \frac{\alpha_H}{\alpha_H+\lambda} P_L$ makes no sense as this already attracts all informed for sure. It cannot price above P_L. Hence, firm L has no profitable deviation.

It remains to verify that all equilibrium objects are well-behaved. First, it is easy to check that $\sigma_H \in (0,1)$ if $R \in (R_0, R_1)$. Second, $\sigma_L > 0$ as $R > R_0 > 1$, while $\sigma_L < 1$ follows from $R < R_1$. Lastly, $F_i(\frac{\alpha_H}{\alpha_H+\lambda} P_L) = 0$, $F_i(P_L) = 1$, and

$$\frac{dF_i(p)}{dp} = \frac{\alpha_H P_H}{p^2[\lambda - \alpha_H(R-1)]} > 0,$$

where the inequality follows from $R < R_1$. □

This completes the proof of Proposition 1. □

Proof of Lemma 1. For high R we are in case C where $p_L^* < p_H^*$. For low R we are in case A where $\sigma_H = 0$ and $\sigma_L > 0$ imply $\mathbb{E}p_L > \mathbb{E}p_H$. For case B we will show that $\mathbb{E}p_L$ strictly increases in P_L, and $\mathbb{E}p_H$ strictly decreases in P_L. Continuity then implies that there must be a unique $P_L \in (P_H/R_1, P_H/R_0)$ where $\mathbb{E}p_L = \mathbb{E}p_H$ which established the result. More precisely, in case B,

$$\begin{aligned}
\mathbb{E}p_L &= \sigma_L P_L + (1-\sigma_L) \int_{\underline{p}}^{P_L} p dF(p) \\
&= \frac{\alpha_H}{1-\alpha_L-\alpha_H} \left[P_H - P_L + P_H \log\left(\frac{1-\alpha_L}{\alpha_H} \frac{P_L}{P_H} \right) \right], \qquad \text{(A.1)}
\end{aligned}$$

while

$$\begin{aligned}
\mathbb{E}p_H &= \sigma_H P_H + (1-\sigma_H) \int_{\underline{p}}^{P_L} p dF(p) \\
&= \frac{P_H \left[\alpha_H(1-\alpha_H) P_H/P_L - \alpha_L(1-\alpha_L) + (1-\alpha_H)\alpha_H \log\left(\frac{1-\alpha_L}{\alpha_H} \frac{P_L}{P_H} \right) \right]}{(1-\alpha_L)(1-\alpha_L-\alpha_H)}. \qquad \text{(A.2)}
\end{aligned}$$

Hence

$$\frac{d\mathbb{E}p_L}{dP_L} = \frac{\alpha_H}{1-\alpha_L-\alpha_H}(P_H/P_L - 1) > 0,$$

and

$$\frac{d\mathbb{E}p_H}{dP_L} = -\frac{P_H}{P_L} \left[\frac{\alpha_H(1-\alpha_H)}{(1-\alpha_L)(1-\alpha_L-\alpha_H)} \right] (P_H/P_L - 1) < 0.$$

□

Proof of Proposition 2. Note that existence follows from Dasgupta and Maskin (1986). We first establish a number of lemmas. First, any firm can always choose to set $p_i = P_i = 1$ and sell to at least its captive consumers. Hence

Lemma A.3. *Each firm has expected profit of at least α_H in equilibrium.*

The firm with the lower P_i sells at most $1-\alpha_H$ at price of at most P_i. If $P_i < \alpha_H/(1-\alpha_H)$, profits are below the α_H it obtains by $p_i = P_i = 1$. Hence

Lemma A.4. *In equilibrium, no firm sets P below $\underline{P}_{min} \equiv \frac{\alpha_H}{1-\alpha_H} > 0$.*

Lemma A.5. *$G(\cdot)$ is atomless.*

Proof. If $G(\cdot)$ has an an atom at P^*, both firms set P^* with some probability $\beta > 0$, yielding profits $\frac{1-\lambda}{2}P^*$. Both firms then prefer to set $P^* - \epsilon$. □

Lemma A.6. *In equilibrium $\overline{P}/\underline{P} > R_0$, so case B can always occur.*

Proof. Suppose to the contrary $\overline{P}/\underline{P} \leq R_0$. Then $\pi_L = \alpha_L P_L$ and $\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}P_L < \pi_L$. With G atomless, setting $\underline{P}$ yields $\alpha_L \underline{P}$. But then, if $\underline{P} > 0$, any $P_i > \underline{P}$ in the equilibrium support yields a lower profit of $\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}\underline{P}$ which cannot be true in equilibrium. Also, $\underline{P} \neq 0$ due to Lemma A.4.

□

Lemma A.7. *$\bar{P} = 1$.*

Proof. Suppose $\bar{P} < 1$. If firm i now deviates to some $P_i \in (\overline{P}, 1]$, it makes either $\alpha_H P_i$ (if the other firm sets $P_j \leq P_i/R_0$ and we are in case B or C) or $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}P_i$ (if $P_j \in (P_i/R_0, \overline{P}]$ and we are in case A). Hence, we can write

$$\pi_i(P_i) = G\left(\frac{P_i}{R_0}\right)\alpha_H P_i + \int_{\frac{P_i}{R_0}}^{\overline{P}} \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P dG(P).$$

Taking the derivative with respect to P_i yields

$$\begin{aligned} \pi_i'(P_i) &= G\left(\frac{P_i}{R_0}\right)\alpha_H + G'\left(\frac{P_i}{R_0}\right)\alpha_H \frac{P_i}{R_0} - \frac{1}{R_0}\left[\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}\right] G'\left(\frac{P_i}{R_0}\right)\frac{P_i}{R_0} \\ &= G\left(\frac{P_i}{R_0}\right)\alpha_H. \end{aligned}$$

Hence, $\lim_{\epsilon \downarrow 0} \pi_i'(\overline{P} + \epsilon) = G(\overline{P}/R_0)\alpha_H > 0$, where the inequality follows from Lemma A.6. But then, setting P marginally above $\overline{P} < 1$ would be a profitable defection, so this cannot be part of an equilibrium. □

Lemma A.8. *There are no gaps in $G(\cdot)$.*

Proof. Suppose G does contain gaps, and the highest is (a, b) for some $a < b < 1$, with $G(a) = G(b) < 1$. From Proposition 1, if $P_j < P_i$, we have that π_i is either $\alpha_H P_i$ (if $P_i/P_j \geq R_0$), or $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_j$ (if $P_i/P_j < R_0$). Hence, conditional on $P_j < P_i$, π_i is weakly increasing in P_i.

If instead $P_i < P_j$, π_i is either $\alpha_L P_i$ (if $P_j/P_i < R_0$), or $\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L} P_j$ (if $R_0 \leq P_j/P_i < R_1$), or $(1 - \alpha_H)P_i$ (if $P_j/P_i \geq R_1$). Again, conditional on $P_j > P_i$, π_i is weakly increasing in P_i. But then we would have $\pi_i(a) < \pi_i(b)$: for $P_i \in (\max\{a, b/R_0\}, b)$, increasing P_i increases π_i when $P_j \in [b, P_i R_0)$, which happens with positive probability since by assumption (a, b) is the highest gap in G. This cannot be the case in equilibrium. □

Taken together, the lemmas above prove Proposition 2. □

Proof of Proposition 3. We first derive the upper bound on profits, then show that these are strictly lower than profits in the Varian (1980) case.

From Proposition 2, both $P = 1$ and $P = 1/R_0$ are in the equilibrium support, and G is atomless. Suppose first firm i sets $P_i = 1$. With $P_j \leq 1$ and $\alpha_H < \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}$, Proposition 1 implies that π_i cannot exceed $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}$. Now suppose firm i sets $P_i = 1/R_0$. If $P_j > P_i$ we are in case A and $\pi_i = \alpha_L P_i$. If $P_j < P_i$, there are two possibilities. In case A of Proposition 1, $\pi_i = \frac{1-\alpha_L}{1-\alpha_H} \alpha_L P_j < \alpha_L P_i$. In case B and C, $\pi_i = \alpha_H P_i < \alpha_L P_i$. Hence, an upper bound on profits is given by $\alpha_L P_i = \frac{\alpha_L}{R_0} = \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}$.

Recall next that with Varian (1980) competition, equilibrium profits are $\pi^* = \frac{1-\lambda}{2}$. For the statement on profits, it thus suffices to show that

$$\min\left\{\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H}, \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}\right\} < \frac{1-\lambda}{2}. \tag{A.3}$$

Using $\alpha_L = \frac{1-\lambda+\mu}{2}$ and $\alpha_H = \frac{1-\lambda-\mu}{2}$, some straightforward algebra implies $\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H} < \frac{1-\lambda}{2}$ if and only if $\lambda < \frac{1+\mu}{3}$, while $\frac{\alpha_H(1-\alpha_H)}{1-\alpha_L} < \frac{1-\lambda}{2}$ if and only if $\lambda > \frac{1-\mu}{3}$. Since at least one of these conditions is always satisfied, (A.3) holds.

□

Proof of Proposition 4. As noted in the main text, for an equilibrium, we need that the right-hand side of (5) is constant for all $P \in [\underline{P}, 1]$.[29] Taking the derivative of (5) with respect to P, collecting terms and simplifying, we thus require for all $P \in [\underline{P}, 1]$ that:

$$G\left(\frac{P}{R_0}\right)\alpha_H - \frac{\alpha_L(\alpha_L - \alpha_H)}{1-\alpha_H} PG'(P) + [G(PR_0) - G(P)]\alpha_L + [1 - G(PR_1)](1-\alpha_H) = 0. \quad \text{(A.4)}$$

We thus have to solve a *functional differential equation*, as $G'(P)$ depends not only on $G(P)$, but also on $G(P/R_0)$, $G(PR_0)$ and $G(PR_1)$. If $P > 1/R_0$, the last two terms of (A.4) collapse to $[1-G(P)]\alpha_L$. If instead $P \in [\underline{P}, \underline{P}R_0]$, the first term of (A.4) vanishes. Hence, the exact differential equation we have to solve depends on the value of P. We thus partition the support $[\underline{P}, 1]$ into a number of intervals. In each interval, $G(P)$ is the solution to a specific differential equation. Differential equations in different intervals may depend on each other.

In the simplest case, the solution consists of three parts. For that, we need in equilibrium that $\underline{P}R_1 \geq 1$ (so that the last term in (A.4) always vanishes) and $\underline{P} > 1/R_0^2$. Equation (A.4) then reduces to a system of three partly interdependent first-order differential equations. Proposition 4 characterizes the solution in that case.

When $\underline{P}R_1 \geq 1$ and $\underline{P} > 1/R_0^2$, we can partition the support into three non-empty intervals $\mathcal{I}_1 = [\underline{P}, 1/R_0)$, $\mathcal{I}_2 = [1/R_0, \underline{P}R_0)$ and $\mathcal{I}_3 = [\underline{P}R_0, 1]$. Denote the distribution function in interval $i \in \{1,2,3\}$ by G_i and the corresponding density function by g_i. Using Proposition 2, we must have that $G_1(\underline{P}) = 0$, $G_1(1/R_0) = G_2(1/R_0)$, $G_2(\underline{P}R_0) = G_3(\underline{P}R_0)$, and $G_3(1) = 1$.

Note that for prices $P \in \mathcal{I}_1$, we have $PR_0 \in \mathcal{I}_3$ and $P/R_0 < \underline{P}$. Hence, using $\underline{P}R_1 \geq 1$, (A.4) then reduces to

$$G_1(P) + kPg_1(P) - G_3(PR_0) = 0, \quad \text{(A.5)}$$

where

$$k \equiv \frac{\alpha_L - \alpha_H}{1-\alpha_H} \in (0,1). \quad \text{(A.6)}$$

[29] It also has to be weakly lower for all $P < \underline{P}$, but this is clearly satisfied as for all $P_i < \underline{P}$, from Proposition 1 the subgame profit $\pi_i(P_i, P_j) = \pi_L(P_i, P_j)$ is weakly increasing in P_i.

For prices $P \in \mathcal{I}_2$, we have $PR_0 > 1$ and $P/R_0 < \underline{P}$, so (A.4) reduces to

$$1 - G_2(P) - kPg_2(P) = 0. \tag{A.7}$$

Finally, for prices $P \in \mathcal{I}_3$, we have $PR_0 > 1$ and $P/R_0 \in \mathcal{I}_1$, so (A.4) reduces to

$$1 - G_3(P) - kPg_3(P) + G_1(P/R_0)\frac{\alpha_H}{\alpha_L} = 0. \tag{A.8}$$

We can now use (A.7) to solve for G_2. Next, (A.8) allows us to write G_1 in terms of G_3 and g_3, and (after differentiation) g_1 in terms of g_3 and g_3'. Plugging these into (A.5) yields a differential equation for G_3 that can be solved analytically. We can then use (A.5) to write G_3 in terms of G_1 and g_1, and (after differentiation) g_3 in terms of g_1 and g_1'. Plugging these into (A.8) yields a differential equation for G_1 that can also be solved analytically.

From (A.7), G_2 has the form

$$G_2(P) = 1 - b_2 P^{-\frac{1}{k}}, \tag{A.9}$$

where b_2 is a coefficient to be determined. To solve for G_1 and G_3, we first introduce the variable $z \equiv P/R_0$, which we substitute in (A.8) to obtain

$$G_1(z) = \frac{\alpha_L}{\alpha_H}\left[kR_0 z g_3(zR_0) - (1 - G_3(zR_0))\right]. \tag{A.10}$$

Taking the derivative with respect to z and simplifying yields

$$g_1(z) = \frac{\alpha_L}{\alpha_H}\left[R_0 g_3(zR_0)(1+k) + kR_0^2 z g_3'(zR_0)\right].$$

Plugging these expressions for G_1 and g_1 into (A.5) yields, after simplification,

$$1 - G_3(P)\left(\frac{\alpha_L - \alpha_H}{\alpha_L}\right) - k(2+k)Pg_3(P) - k^2P^2g_3'(P) = 0. \tag{A.11}$$

We conjecture that $G_3(P)$ has the following functional form:

$$G_3(P) = a_3 + b_{3,1}P^{c_{3,1}} + b_{3,2}P^{c_{3,2}}, \tag{A.12}$$

such that

$$g_3(P) = b_{3,1}c_{3,1}P^{c_{3,1}-1} + b_{3,2}c_{3,2}P^{c_{3,2}-1} \tag{A.13}$$

and

$$g_3'(P) = b_{3,1}c_{3,1}(c_{3,1}-1)P^{c_{3,1}-2} + b_{3,2}c_{3,2}(c_{3,2}-1)P^{c_{3,2}-2}.$$

Substituting these expressions into (A.11) and comparing coefficients with (A.12), we find that

$$a_3 = \frac{\alpha_L}{\alpha_L - \alpha_H} \tag{A.14}$$

and

$$c_{3,(\cdot)} = -\frac{1}{k}\left(1 \pm \sqrt{\frac{\alpha_H}{\alpha_L}}\right),$$

while $b_{3,1}$ and $b_{3,2}$ are still undetermined.

Note that $c_{3,1}$ and $c_{3,2}$ are given by the two solutions to the quadratic equation $k^2c_3^2 + 2kc_3 + \frac{\alpha_L-\alpha_H}{\alpha_L} = 0$. Necessarily, it must hold that $c_{3,1} \neq c_{3,2}$: otherwise, we would have that $G_3(P)$ is of the form $a_3 + b_3P^{c_3}$, which cannot constitute the general solution to a second-order ordinary differential equation. For concreteness, in what follows let

$$c_{3,1} = -\frac{1}{k}\left(1 - \sqrt{\frac{\alpha_H}{\alpha_L}}\right) \tag{A.15}$$

$$c_{3,2} = -\frac{1}{k}\left(1 + \sqrt{\frac{\alpha_H}{\alpha_L}}\right). \tag{A.16}$$

Using that $G_3(1) = 1$, equation (A.12) finally gives us the requirement

$$b_{3,2} = 1 - a_3 - b_{3,1}. \tag{A.17}$$

We next introduce the variable $q \equiv PR_0$, which we substitute into (A.5) to obtain

$$G_3(q) = G_1\left(\frac{q}{R_0}\right) + \frac{k}{R_0}qg_1\left(\frac{q}{R_0}\right).$$

After taking the derivative with respect to q, we obtain

$$g_3(q) = \frac{k+1}{R_0}g_1\left(\frac{q}{R_0}\right) + \frac{k}{R_0^2}qg_1'\left(\frac{q}{R_0}\right).$$

Plugging these expressions for G_3 and g_3 into (A.8) and simplifying yields

$$1 - G_1(P)\left(\frac{\alpha_L-\alpha_H}{\alpha_L}\right) - k(2+k)Pg_1(P) - k^2P^2g_1'(P) = 0.$$

This differential equation perfectly coincides with that for $G_3(P)$ above. Hence, it follows that $G_1(P)$ must take the functional form

$$G_1(P) = a_3 + b_{1,1}P^{c_{3,1}} + b_{1,2}P^{c_{3,2}}, \tag{A.18}$$

with a_3, $c_{3,1}$ and $c_{3,2}$ as specified in equations (A.14), (A.15) and (A.16) above, while $b_{1,1}$, $b_{1,2}$ are yet undetermined coefficients.

Plugging (A.12) and (A.13) (with G_3 and g_3 evaluated at $\underline{P}R_0$) into (A.10), some rearranging and simplifying reveals that $G_1(P)$ must be of the form

$$G_1(P) = a_3 + \left[b_{3,1}R_0^{c_{3,1}}\sqrt{\frac{\alpha_L}{\alpha_H}}\right]P^{c_{3,1}} + \left[-b_{3,2}R_0^{c_{3,2}}\sqrt{\frac{\alpha_L}{\alpha_H}}\right]P^{c_{3,2}}.$$

Comparing this with (A.18), we can pin down $b_{1,1}$ and $b_{1,2}$ as functions of $b_{3,1}$ (using that $b_{3,2} = 1 - a_3 - b_{3,1}$):

$$b_{1,1}(b_{3,1}) = b_{3,1}R_0^{c_{3,1}}\sqrt{\frac{\alpha_L}{\alpha_H}}, \tag{A.19}$$

$$b_{1,2}(b_{3,1}) = -(1 - a_3 - b_{3,1})R_0^{c_{3,2}}\sqrt{\frac{\alpha_L}{\alpha_H}}. \tag{A.20}$$

The requirement that $G_1(1/R_0) = G_2(1/R_0)$ next pins down b_2 as a function of $b_{3,1}$:

$$b_2(b_{3,1}) = \frac{(1 - a_3)\left(1 + \sqrt{\frac{\alpha_L}{\alpha_H}}\right) - 2b_{3,1}\sqrt{\frac{\alpha_L}{\alpha_H}}}{R_0^{\frac{1}{k}}}. \tag{A.21}$$

Finally, the requirement that $G_2(\underline{P}R_0) = G_3(\underline{P}R_0)$ yields an expression for $b_{3,1}$, conditional on $\underline{P}$:

$$1 - b_2(b_{3,1}) \cdot (\underline{P}R_0)^{-\frac{1}{k}} = a_3 + b_{3,1}(\underline{P}R_0)^{c_{3,1}} + (1 - a_3 - b_{3,1})(\underline{P}R_0)^{c_{3,2}}.$$

As this is linear in $b_{3,1}$, we can express $b_{3,1}$ directly as a function of $\underline{P}$:

$$b_{3,1}(\underline{P}) = \frac{(1 - a_3)\left[R_0^{1/k}(\underline{P}R_0)^{1/k}(1 - (\underline{P}R_0)^{c_{3,2}}) - \left(1 + \sqrt{\frac{\alpha_L}{\alpha_H}}\right)\right]}{R_0^{1/k}(\underline{P}R_0)^{1/k}\left[(\underline{P}R_0)^{c_{3,1}} - (\underline{P}R_0)^{c_{3,2}}\right] - 2\sqrt{\frac{\alpha_L}{\alpha_H}}}. \tag{A.22}$$

The potential equilibrium value of $\underline{P}$ can then be found by solving the consistency requirement

$$G_1(\underline{P}) = a_3 + \left[b_{3,1}(\underline{P})R_0^{c_{3,1}}\sqrt{\frac{\alpha_L}{\alpha_H}}\right]\underline{P}^{c_{3,1}} + \left[-(1-a_3-b_{3,1}(\underline{P}))R_0^{c_{3,2}}\sqrt{\frac{\alpha_L}{\alpha_H}}\right]\underline{P}^{c_{3,2}} = 0. \tag{A.23}$$

Hence, $\underline{P}$ is implicitly defined by

$$a_3 + b_{1,1}(\underline{P})\underline{P}^{c_{3,1}} + b_{1,2}(\underline{P})\underline{P}^{c_{3,2}} = 0. \tag{A.24}$$

Now, if it indeed holds that $\underline{P}R_1 \geq 1$ and $\underline{P} > 1/R_0^2$, as assumed throughout in this proof, and furthermore $G_1(P)$, $G_2(P)$ and $G_3(P)$ are all strictly increasing in P, then a valid solution has been found. If this is not the case, then no equilibrium with the postulated structure exists for the considered parameter combination.

For any parameter combination (λ, μ), we can thus try to find a numerical solution as follows. First, we use (A.24) to solve numerically for $\underline{P}$. If this satisfies $\underline{P} \in (1/R_0^2, 1/R_0)$ and $\underline{P}R_1 \geq 1$, we can determine $G(P)$ using (6). If not, an equilibrium of this particular form fails to exist.

Note lastly that the expressions in the proposition are obtained by letting $a \equiv a_3$, $\kappa_1 \equiv c_{3,1}$ and $\kappa_2 \equiv c_{3,2}$.

Figure A.2 shows for which parameter combinations the above procedure indeed yields a valid solution for the first-stage price distribution. In particular, this reveals that the procedure works for all λ sufficiently large ($\lambda \gtrapprox 0.38$), irrespective of μ. □

Proof of Lemma 2. We first establish the following:

Lemma A.9. *If $\tilde{\theta}$ exists, it is such that $\tilde{\alpha}_H$ is a root of*

$$h(\tilde{\alpha}_H; R) = \left(1 - \frac{1-\tilde{\alpha}_H}{\lambda+\tilde{\alpha}_H}\right)\log\left(\frac{\lambda+\tilde{\alpha}_H}{\tilde{\alpha}_H R}\right) + \frac{1-\lambda}{\tilde{\alpha}_H} - \frac{1}{R} - \frac{1-\tilde{\alpha}_H}{\lambda+\tilde{\alpha}_H}R.$$

Moreover, we have (1) $\tilde{\theta} > 1/2$, (2) $\frac{\partial h}{\partial \tilde{\alpha}_H} < 0$, (3) $\frac{\partial h}{\partial R} < 0$, and (4) $\frac{\partial^2 h}{\partial \tilde{\alpha}_H \partial R} > 0$.

Proof. As noted, we need $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$. Equating (A.1) and (A.2) in the proof of Lemma 1 and simplifying, we thus need $\tilde{\theta}$ to be such that

$$1 - \frac{1}{R} - \frac{\tilde{\alpha}_L - \tilde{\alpha}_H}{1-\tilde{\alpha}_L}\log\left(\frac{1-\tilde{\alpha}_L}{\tilde{\alpha}_H}\frac{1}{R}\right) - \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L}R + \frac{\tilde{\alpha}_L}{\tilde{\alpha}_H} = 0.$$

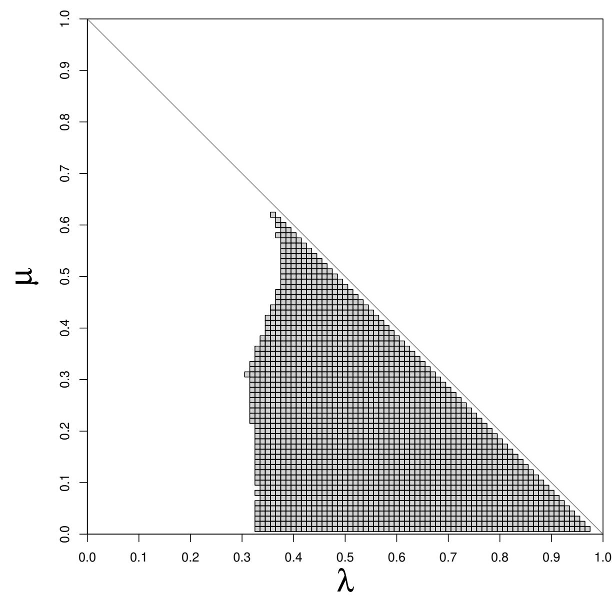


Figure A.2: Parameter combinations ($\lambda \in \{0.01, 0.02, ..., 0.97\}$, $\mu \in \{0.01, 0.02, ..., 0.97\}$, $\lambda + \mu \leq 0.98$). Each square corresponds to a feasible parameter combination, centered at the respective parameters. Black squares indicate parameter combination for which Proposition 4 yields a valid solution.

Using $\tilde{\alpha}_L = 1 - \lambda - \tilde{\alpha}_H$ yields the expression for $h(\tilde{\alpha}_H; R)$.

To prove (1), we show that $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ for $\theta \leq \frac{1}{2}$. In case C of Proposition 1, this is true as both firm charge the list price. In case B,

$$\mathbb{E}p_L(\theta) = (1 - \sigma_L) \int_{\underline{p}}^{P_L} pdF(p) + \sigma_L P_L$$
$$\mathbb{E}p_H(\theta) = (1 - \sigma_H) \int_{\underline{p}}^{P_L} pdF(p) + \sigma_H P_H.$$

With $P_H > P_L$, if $\sigma_H \geq \sigma_L$, we have $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$. Now $\sigma_H \geq \sigma_L$ requires

$$(1 - \tilde{\alpha}_H)\tilde{\alpha}_H R - (1 - \tilde{\alpha}_L)\tilde{\alpha}_L \geq (1 - \tilde{\alpha}_L)\tilde{\alpha}_H R - (1 - \tilde{\alpha}_L)\tilde{\alpha}_H,$$

which implies

$$(\tilde{\alpha}_L - \tilde{\alpha}_H)\tilde{\alpha}_H R \geq (\tilde{\alpha}_L - \tilde{\alpha}_H)(1 - \tilde{\alpha}_L).$$

With $\theta \le 1/2$, we have $\tilde{\alpha}_L - \tilde{\alpha}_H \le 0$, so this implies

$$R \le \frac{1-\tilde{\alpha}_L}{\tilde{\alpha}_H} = \tilde{R}_1,$$

which is true since we are in Case B.

To prove the other claims, note that

$$\frac{\partial h}{\partial R} = -\frac{(R-1)\,(R\,(1-\alpha_H)+\lambda+\alpha_H)}{R^2\,(\lambda+\alpha_H)} < 0$$

and hence

$$\frac{\partial^2 h}{\partial \tilde{\alpha}_H \partial R} = \frac{(1+\lambda)\,(R-1)}{R\,(\lambda+\tilde{\alpha}_H)^2} > 0,$$

which establishes claims (3) and (4). Note next that

$$\frac{\partial h}{\partial \tilde{\alpha}_H} = \frac{\lambda-\lambda^2-2\lambda\tilde{\alpha}_H}{\tilde{\alpha}_H\,(\lambda+\tilde{\alpha}_H)^2} + \left(\ln\frac{\lambda+\tilde{\alpha}_H}{\tilde{\alpha}_H R}\right)\frac{1+\lambda}{(\lambda+\tilde{\alpha}_H)^2} - \frac{1}{\tilde{\alpha}_H^2}\,(1-\lambda) + R\frac{1+\lambda}{(\lambda+\tilde{\alpha}_H)^2}.$$

Claim (4) then implies that if $\partial h/\partial\tilde{\alpha}_H$ is negative at R_1, then it is negative for all $R \in (R_0, R_1)$. We thus need[30]

$$\left.\frac{\partial h}{\partial \tilde{\alpha}_H}\right|_{R=R_1} = \frac{\lambda-\lambda^2-2\tilde{\alpha}_H\lambda}{(\lambda+\tilde{\alpha}_H)^2} - \frac{1-\lambda}{\tilde{\alpha}_H} + \frac{1+\lambda}{\lambda+\tilde{\alpha}_H} < 0.$$

Multiplying by $\tilde{\alpha}_H(\lambda+\tilde{\alpha}_H)^2$, we require

$$\tilde{\alpha}_H(\lambda-\lambda^2-2\tilde{\alpha}_H\lambda) - (1-\lambda)(\lambda+\tilde{\alpha}_H)^2 + \tilde{\alpha}_H(1+\lambda)(\lambda+\tilde{\alpha}_H) < 0,$$

which simplifies to

$$(2\tilde{\alpha}_H - 1) + \lambda < 0.$$

Using $\lambda = 1-\tilde{\alpha}_L-\tilde{\alpha}_H$, this simplifies to $\tilde{\alpha}_H < \tilde{\alpha}_L$ which is true for $\tilde{\theta} > \frac{1}{2}$. □

To establish Lemma 2, note that from the proof of Lemma A.9, we have $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ if $\theta < \frac{1}{2}$. By construction $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$. Since $\mathbb{E}p_L(\theta)$ and $\mathbb{E}p_H(\theta)$ are continuous in θ, this establishes existence.

[30] Note that the term containing the logarithm drops at $R = R_1$.

For uniqueness $\mathbb{E}p_L - \mathbb{E}p_H$ needs to be monotonic in θ for $\theta \in (\frac{1}{2}, 1)$. Now

$$\frac{dh}{d\tilde{\theta}} = \frac{dh}{d\tilde{\alpha}_H}\frac{d\tilde{\alpha}_H}{d\tilde{\theta}} = -\mu\frac{dh}{d\tilde{\alpha}_H},$$

where we use $d\tilde{\alpha}_H/d\tilde{\theta} = -\mu$. Hence it is sufficient to have that h is monotonic in $\tilde{\alpha}_H$, which is true from Claim 2 in Lemma A.9. □

Proof of Proposition 5. We check whether both firms can set the same P in equilibrium. This would imply per-firm profits of $\frac{1-\lambda}{2}P$. We proceed as follows:

1. Suppose i deviates to a lower P_i with $P/P_i \geq R^*$. From Lemma 1, we then have $\mathbb{E}p_L < \mathbb{E}p_H$, so all partially informed visit i. This yields profits

$$\Pi_i^d(P_i; P) = \begin{cases} (1-\alpha_H)P_i & \text{if} \quad P_i \leq P/R_1 \\ \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P & \text{if} \quad P_i \in (P/R_1, P/R^*]. \end{cases} \tag{A.25}$$

With $(1-\alpha_H)P_i$ strictly increasing in P_i, it is never a best reply to set $P_i < P/R_1$. Hence, the best possible defection in this range yields $\Pi_i^d = \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P$. This is weakly lower than $\frac{1-\lambda}{2}P$ whenever $\lambda \geq \frac{1-\mu}{3}$. Hence, for $\lambda \geq \frac{1-\mu}{3}$, firm i weakly prefers $P_i = P$ over any $P_i \leq P/R^*$.

2. Suppose i deviates to a lower P_i with $P/P_i < R^*$. From Lemma 1, not all partially informed consumers go to i. Moreover, i and j must have the same expected retail price. From (7), this yields

$$\Pi_i^d(P_i; P) = \frac{(1-\tilde{\alpha}_H)\tilde{\alpha}_H}{1-\tilde{\alpha}_L}P = \frac{(1-\tilde{\alpha}_H)\tilde{\alpha}_H}{\tilde{\alpha}_H + \lambda}P.$$

This implies that

$$\frac{\partial \Pi_i^d(P_i; P)}{\partial P_i} = \frac{\partial \Pi_i^d(P_i; P)}{\partial \tilde{\alpha}_H}\frac{d\tilde{\alpha}_H(P_i)}{dP_i} = -\left[1 - \frac{(1+\lambda)\,\lambda}{(\lambda + \tilde{\alpha}_H)^2}\right]P\frac{d\tilde{\alpha}_H}{dP_i}. \tag{A.26}$$

Using the implicit function theorem,

$$\frac{d\tilde{\alpha}_H}{dR} = -\frac{\partial h/\partial R}{\partial h/\partial \tilde{\alpha}_H} < 0,$$

as follows from claims (2) and (3) of Lemma A.9. With $R = P/P_i$,

$$\frac{d\tilde{\alpha}_H}{dP_i} = \frac{d\tilde{\alpha}_H}{dR}\frac{dR}{dP_i} > 0.$$

Hence, $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} \geq 0$ for all $P_i \in (P/R^*, P)$ if the bracketed term in (A.26) is weakly negative in this interval. This term strictly increases in $\tilde{\alpha}_H$, which strictly increases in P_i. We thus need $\lim_{P_i \to P}\left[1 - \frac{(1+\lambda)\lambda}{(\lambda+\tilde{\alpha}_H)^2}\right] \leq 0$. As $\lim_{P_i \to P} \tilde{\alpha}_H = \frac{1-\lambda}{2}$, this is equivalent to

$$1 - \frac{4\,(1+\lambda)\,\lambda}{(1+\lambda)^2} \leq 0,$$

which reduces to $\lambda \geq 1/3$. Hence, if $\lambda \geq 1/3$, we have $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} \geq 0$ for $P_i \in (P/R^*, P)$, so i sets $P_i = P$. If $\lambda < 1/3$, we have $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} < 0$ for all P_i sufficiently close below P, so firm i undercuts P.

3. Suppose $P < 1$ and firm i deviates to a higher P_i with $P_i/P < R^*$. That yields $\Pi_i^d(P_i;P) = \tilde{\alpha}_H P_i$, so

$$\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} = \frac{d\tilde{\alpha}_H}{dP_i}P_i + \tilde{\alpha}_H = \frac{d\tilde{\alpha}_H}{dR}\frac{dR}{dP_i}P_i + \tilde{\alpha}_H = -\frac{\partial h/\partial R}{\partial h/\partial \tilde{\alpha}_H}\frac{P_i}{P} + \tilde{\alpha}_H.$$

Evaluated at $P_i = P$, the first term is zero, hence

$$\left.\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i}\right|_{P_i=P} = \frac{1-\lambda}{2} > 0.$$

4. For $\lambda \geq 1/3$, steps 1 and 2 imply that $P = 1$ is an equilibrium, while step 3 implies that an equilibrium cannot have $P < 1$.

5. For $\lambda < 1/3$, step 2 implies that any firm wants to deviate from any symmetric equilibrium.

□

Proof of Proposition 6. The case $\lambda \geq 1/3$ follows from Proposition 5. For $\lambda < 1/3$, first note that with rational consumers we can never end up in case A of Proposition 1, as Lemma 1 implies that in case A we always have $\mathbb{E}p_L > \mathbb{E}p_H$. Hence only cases B and C are relevant.

Suppose that P_L and P_H are such that $R < R^*$. From Proposition 1,

$$\begin{aligned} \pi_L + \pi_H &= \tilde{\alpha}_H P_H \left(1 + \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L}\right) \\ &\leq \tilde{\alpha}_H \left(1 + \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L}\right) = \frac{(1+\lambda)[1-\lambda-\mu(2\tilde{\theta}-1)]}{1+\lambda-\mu(2\tilde{\theta}-1)}, \end{aligned} \quad (A.27)$$

where $\tilde{\theta}$ again equalizes expected retail prices. The right-hand side decreases in $\tilde{\theta}$. Hence, an upper bound can be found by setting $\tilde{\theta} = 1/2$. This implies

$$\pi_L + \pi_H \leq 1 - \lambda.$$

Now suppose P_L and P_H are such that $R \in [R^*, R_1)$. From Proposition 1, (A.27) then still applies, but with $\tilde{\theta} = 1$ (and hence α_H and α_L rather than $\tilde{\alpha}_H$ and $\tilde{\alpha}_L$, respectively). Hence, we now have, $\pi_L + \pi_H < 1 - \lambda$.

Finally, suppose P_L and P_H are such that $R \geq R_1$. From Proposition 1,

$$\begin{aligned} \pi_L + \pi_H &= (1-\alpha_H)P_L + \alpha_H P_H \\ &\leq (1-\alpha_H)\frac{P_H}{R_1} + \alpha_H P_H = \alpha_H P_H \left(1 + \frac{1-\alpha_H}{1-\alpha_L}\right) \\ &\leq \alpha_H \left(1 + \frac{1-\alpha_H}{1-\alpha_L}\right) < 1 - \lambda, \end{aligned}$$

where the last inequality again follows from the same argument used for (A.27). Without list prices, we are in the Varian case and total profits equal $1 - \lambda$. This establishes the result. □

Proof of Proposition 8. With $\mu \leq 2\lambda$, the optimal defection is in the retail-pricing stage. The result then follows immediately from Proposition 3. For $\mu > 2\lambda$, the optimal defection is in the list-price stage, with $\pi^D = \frac{1-\lambda+\mu}{2} = \alpha_L$. We thus need to establish that $\overline{\delta} = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} < 1/2$, which is equivalent to

$$\pi^N < 2\pi^C - \pi^D = 1 - \alpha_L.$$

This is indeed true: from Proposition 3 we know that $\pi^N < \frac{\alpha_L(1-\alpha_L)}{1-\alpha_H} < 1 - \alpha_L$, where the last inequality follows from $\frac{\alpha_L}{1-\alpha_H} < 1$. □

Appendix B Numerical analysis

B.1 Numerical solution of the baseline model

Our numerical approach proceeds as follows. For any (λ, μ), we discretize the action space by breaking down the candidate support $[\underline{P}_{min}, 1]$ into l actions $a_1, ..., a_l$, where a_k $(k \in \{1, ..., l\})$ implies choosing $P = \underline{P}_{min} + (k-1)\left(\frac{1-\underline{P}_{min}}{l-1}\right)$. We then use Proposition 1 to construct a $l \times l$ payoff matrix A, with a_{ij} i's expected profit when choosing a_i while j chooses a_j. We set $a_{ii} = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} a_i$ on the main diagonal. Hence the row player is treated as having a strictly higher list price in case of a tie. This slightly increases the incentives to compete, but improves accuracy by creating just a single discontinuity in payoffs around $a_i = a_j$.

Let f_k denote the $(l-k+1) \times 1$ vector describing the frequency distribution of actions $(a_k, ..., a_l)$. Let ι_k denote a vector of ones of the corresponding length. Finally, let A_k be the $(l-k+1) \times (l-k+1)$ submatrix of A with rows k to l and columns k to l. Then, for given k, the following linear system in f_k is a candidate equilibrium with expected profit γ:

$$A_k f_k = \gamma \cdot \iota_k \tag{B.1}$$

$$\iota_k' f_k = 1. \tag{B.2}$$

Here a_k serves as a guess for the lower bound $\underline{P}$ of $G(P)$. Equation (B.1) then states that for given support $\{a_k, ..., a_l\}$, each action yields the same payoff γ (as $G(P)$ cannot contain gaps), while (B.2) requires frequencies to sum to one.

To numerically approximate the equilibrium, we use the following algorithm. First, take $k = 1$. Second, solve the above linear system of $l-k+2$ equations in $l-k+2$ unknowns for f_k and γ. If A_k is invertible and $\iota_k' A_k^{-1} \iota_k \neq 0$ a unique solution exists and is given by[31]

$$\gamma = \frac{1}{\iota_k' A_k^{-1} \iota_k} \tag{B.3}$$

$$f_k = \frac{A_k^{-1} \iota_k}{\iota_k' A_k^{-1} \iota_k}. \tag{B.4}$$

[31] To see this, note that we may first multiply (B.1) by $\iota_k' A_k^{-1}$ from the left (if A_k is invertible), resulting in $\iota_k' f_k = \gamma \cdot \iota_k' A_k^{-1} \iota_k$. Substituting $\iota_k' f_k$ from (B.2) and dividing through $\iota_k' A_k^{-1} \iota_k$ yields (B.3). Plugging this back into $f_k = \gamma \cdot A_k^{-1} \iota_k$ (as obtained from (B.1)) gives f_k.

If $f_k > 0$, we have a solution. If not, we increase k by 1 and repeat the procedure. The fact that $\underline{P} < 1/R_0$ yields another robustness check: the algorithm should terminate for some k with $a_k < 1/R_0$. Otherwise, it fails to find the equilibrium.

Figure B.2 gives an example for $\lambda = 0.4$ and $\mu = 0.2$. For these values, we can also use Proposition 4 to check the performance of our numerical procedure. With $l = 201$ grid points, our algorithm stops at $k = 78$ for an estimated lower bound of $\underline{P} = 0.53875$. The frequency distribution appears to consist of three different parts, with transitions around $0.67 \approx 1/R_0$ and $0.81 \approx \underline{P}R_0$.[32] This is also implied by Proposition 4. Figure B.3 shows the corresponding CDF.

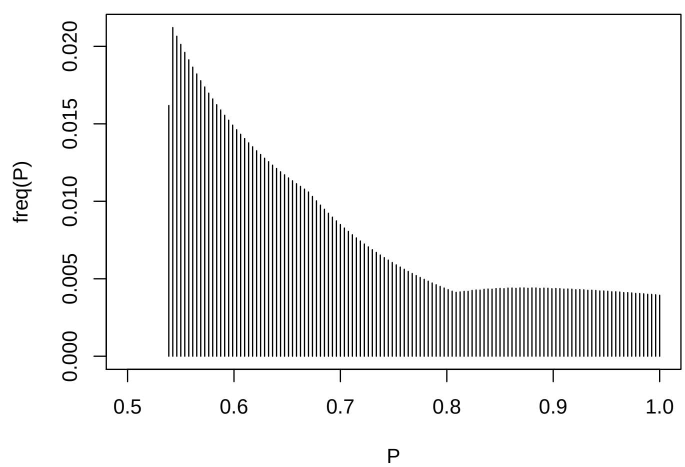


Figure B.2: Approximated equilibrium PDF ($\lambda = 0.4$, $\mu = 0.2$).

B.2 Approximated equilibrium list price CDFs with rational consumers

To illustrate the equilibrium complexity with rational partially informed consumers when $\lambda < 1/3$, Figure B.4 shows the approximated equilibrium

[32]The apparent discontinuity between the first and second price is an artifact of the discretization. It vanishes as the grid size l increases.

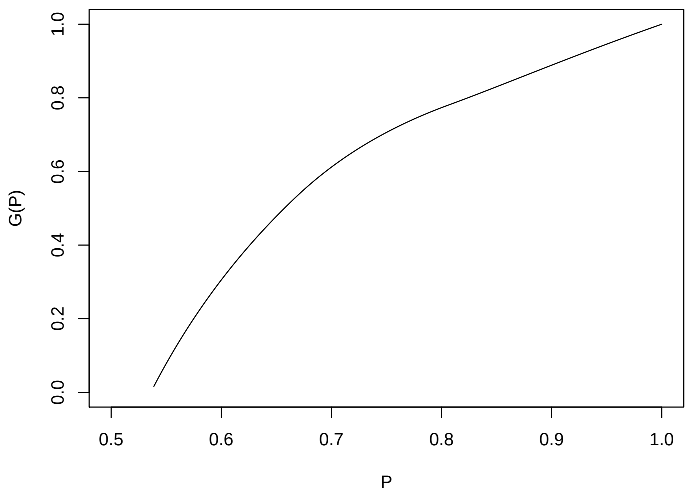


Figure B.3: Approximated equilibrium CDF ($\lambda = 0.4$, $\mu = 0.2$).

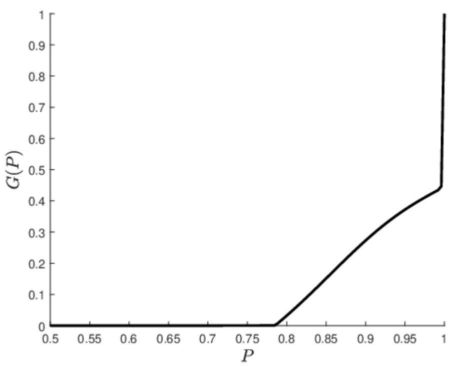
CDF $G(P)$ for four sets of parameters. The equilibrium in the top-left panel is fairly well-behaved, but does have a mass point at $P = 1$. The equilibrium in the top-right panel has two mass points: one at $P = 1$ and one at $P \approx 0.55$. Moreover, the support has a gap in the range $[0.65, 1)$. In the bottom-left panel, there are two gaps but only one mass point. The bottom-right panel has two mass points and two gaps.

Even though the parameter values are relatively close, the resulting equilibria are qualitatively quite different. The likely cause is that small changes in parameter values may trigger substantial differences in best replies, as we saw in the discussion of Figure 5. From our numerical results, even with $\lambda < 1/3$, firms refrain from using effective list prices (and hence set $P = 1$) with positive probability.

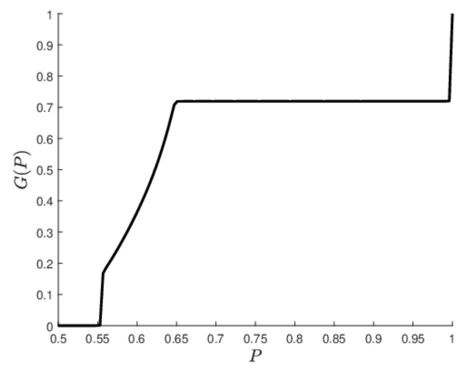
B.3 Details to Numerical Result 2

Figure B.5 gives the numerically approximated differences in expected equilibrium profits between the case of rational and that of myopic partially informed consumers. In line with Numerical Result 2, this difference is always positive.

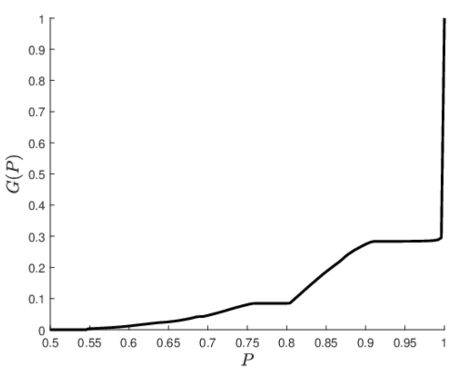
Figure B.4: Approximated first-stage equilibrium CDFs with rational partially informed consumers.



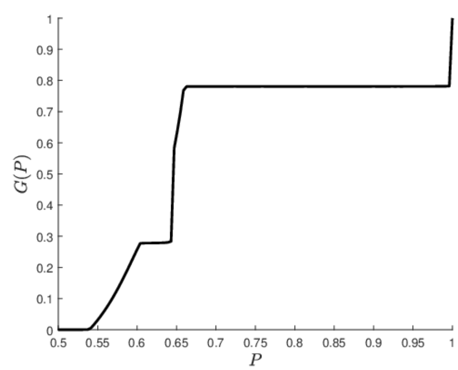
(a) $\lambda = 0.25, \mu = 0.3$



(b) $\lambda = 0.2, \mu = 0.3$



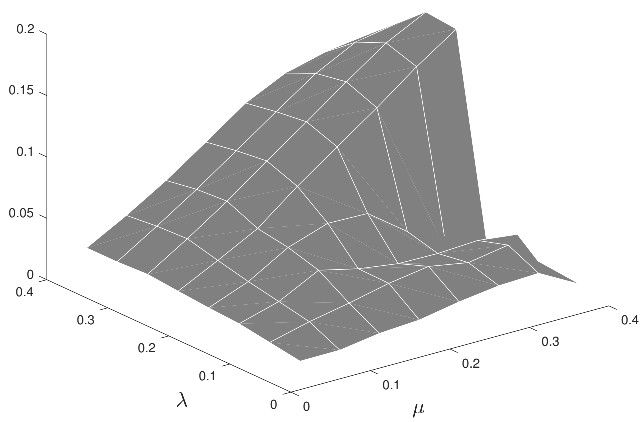
(c) $\lambda = 0.25, \mu = 0.15$



(d) $\lambda = 0.15, \mu = 0.3$

Approximated first-stage equilibrium CDFs for various parameter combinations. In each case, an equidistant grid of size 256 over the interval $[0, 1]$ was used.

Figure B.5: Equilibrium profit differences between the rational and the myopic case.



For both cases, profits are approximated by solving for the Nash equilibrium of a discretized version of the game. The discretization uses an equidistant grid of size 256 over the interval $[0, 1]$. Positive values mean that profits are higher in the case of rational consumers.