Winning back the unfaithful while exploiting the loyal

Retention offers and heterogeneous switching costs

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Abstract

We study retention offers, the practice that firms lower prices to consumers that want to cancel their contract. In a two-period Hotelling model, consumers have either low or high switching costs. In the second period, firms try to poach consumers. Consumers with a poaching offer can solicit a retention offer from their original supplier. In equilibrium, only low switching costs go though the effort of obtaining a poaching offer. Hence, retention offers serve as a mechanism to price discriminate against high switching cost consumers. In our model, the possibility of retention offers increases prices and profits. Consumer surplus decreases.

Keywords: Switching costs, retention offers, behavior-based price discrimination, poaching.

JEL classification: D11, D43, L13.

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1 Introduction

In subscription-type markets, e.g. those for credit cards, cable, telecom, and insurance, firms are often willing to offer a better deal to consumers who indicate that they want to cancel their subscription. These offers are known as retention offers, as firms make them in an attempt to retain fickle consumers. Consumers' reactions to these practices differ. Some seem largely unaware of it, or at least unwilling to exploit such offers. Others actively chase them, sharing details of current offers on websites like flyertalk.com.

In this paper, we analyze retention offers. We assume that there are two types of consumers; those with relatively low, and those with relatively high switching costs. Firms can use retention offers to screen consumers with low switching costs. Consumers that have already gone through the trouble of obtaining an offer from a competing firm, signal that they have low switching costs and hence are likely to switch. Retention offers then effectively serve as a mechanism to price discriminate against consumers with high switching costs.

We thus focus on cases where consumers cancel their current subscription in favor of a competitor. For example, in the UK, Ofcom (2010) reports that in e.g. mobile telephony, consumers that want to switch have to contact their current provider and request a code which they must communicate to their new provider to complete the switch. However, when applying for such a code, the current provider can, and often does, make a retention offer. Indeed, this paper was inspired by a similar experience of one of the authors. After having switched to a cheaper car insurance, he still received a renewal from the old insurer. He phoned them, the company apologized, asked why he cancelled his policy, and what price the new insurer charged. It then offered a price slightly below that – which he was willing to accept. It is exactly this experience that we try to model in this paper.

We study a two-period model with two firms located at the endpoints of a Hotelling line. In the second period, firms set prices based on buying behavior in period 1. In particular, firm B can try to poach consumers from firm A by charging them a lower price. Once a consumer indicates that she intends to switch from A to B to take advantage of that poaching price, however, firm A can make a retention offer. In the equilibrium of our model, low-switching-cost consumers strategically solicit offers from the competing firm to secure a retention offer from their current provider – even if they have no intention to switch. Soliciting offers requires costly effort, and high-switching-cost consumers do not find making that effort worthwhile. Hence, using retention offers allows firms to price discriminate between the two types.

We find that the possibility of retention offers increases prices. Prices for loyal consumers increase, as this pool of consumers is less likely to switch on average. But poaching prices increase as well; as low-cost consumers have already incurred part of their switching costs, they become easier to poach. Equilibrium prices in the first period also increase. As competition for consumers with low switching costs is fiercer in the second period, firms are less eager to attract these consumers in period 1. The welfare effects are ambiguous. Firms are better off, while consumers are worse off. The latter applies to all individual high-switching-cost consumers, and to consumers as a whole. The effect on individual low-cost consumers is ambiguous.

This paper clearly fits in the literature on behavior-based price discrimination. Classic references in this field include Chen (1997), Fudenberg and Tirole (2000) and Taylor (2003), that all look at multi-period models in which firms can base the price they charge on a consumer's purchase history. Chen and Pearcy (2010) allow consumer tastes to evolve over the course of the game. Gehrig, Shy and Stenbacka (2011) study the welfare effects of behavior-based price discrimination in the context of entry deterrence. Yet, none of these papers allows for retention offers. Our paper adds to the literature on switching costs, of which overviews can be found in Klemperer (1995) and Farrell and Klemperer (2007).

Esteves (2014) studies a model similar to ours. She extends Fudenberg and Tirole (2000) with retention offers. Crucially, however, her consumers do not strategically solicit an offer from the competing firm, in an attempt to obtain a better deal from their current supplier. In Esteves (2014), consumers do not rationally foresee that retention offers will be made, hence they do not affect first-period prices.

Finally note that retention offers differ from price-matching policies, in which a supplier is always willing to match a lower price of a competitor. Such price-matching policies do not depend on purchase behavior of consumer. Also, in our model, we will see that the equilibrium retention price is actually higher than the poaching price offered by the competitor, simply because the consumer has already revealed a preference for this supplier by her past buying behavior. Price-matching policies are studied in e.g. Arbatskaya, Hviid and Shaffer (2004) and Corts (1997).

This paper is organized as follows. First, section 2 introduces the model. Section 3 considers a benchmark in which there are no retention offers, but there is poaching and heterogeneous switching costs. The model with retention offers is studied in section 4. We study the effects of the possibility of retention offers in Section 5 and conclude in Section 6.

2 The model

A unit mass of consumers is uniformly distributed on a Hotelling line. Transportation costs are normalized to 1. Firms A and B are located at 0 and 1 respectively and face marginal costs c. There are 2 periods. Consumers have unit demand in each period, and willingness-to-pay r, gross of transportation costs. The market is fully covered. Firms and consumers use a common discount factor $\delta \in (0, 1)$.

We have two types of consumers: those with high switching costs z_H, and those with low switching costs $z_L < z_H$. The share of low types is given by $\lambda \in (0, 1)$, independent of location. Switching costs are incurred if a consumer switches suppliers in period 2.

In the second period, a consumer has a number of options. First, she can choose to simply stick to her current supplier. Second, she may solicit a retention offer by contacting her current supplier and threatening to leave – only to decide not to switch anyway. Doing so involves some costs z_i^1 for a type i consumer, with $z_L^1 < z_L^2$. Hence z_i^1 are the **solicitation costs**, the costs to a type i of securing a retention offer. Third, a consumer can secure a retention offer and then decide to switch anyhow. Total costs in that case equal $z_i^1 + z_i^2$: here, z_i^1 are again the solicitation costs of securing a retention offer, and z_i^2 the **effectuation costs**: the additional costs of effectuating a switch that come on top of the costs of soliciting a retention offer, with $z_L^2 < z_H^2$. Finally, a consumer a choose to switch suppliers right away, without first securing a retention offer. As noted, the costs of doing so equal z_i, with $z_L < z_H$. Crucially, we thus assume that in any scenario, the costs of a low type are lower that those of a high type.

For simplicity, we will assume that $z_i^1 + z_i^2 = z_i$, for $i = 1, 2$. Hence, we assume that the total costs of making a switch are independent of whether a consumer solicits a retention offer. This is the case, for example, if a firm always makes a retention offer to a consumer that wants to cancel her service, even if she does not ask for it.[1] One could also argue that sometimes, trying to obtain a retention offer does involve additional costs. In that case, we would have $z_i^1 + z_i^2 > z_i$. While greatly complicating the analysis, allowing for this would not affect the gist of it.[2]

The timing of the game is as follows. In **period 1**, A and B simultaneously set prices p_A^1 and p_B^1, respectively. Consumers observe these prices, and decide where to buy in the first period. A fraction $\hat{x}_i^1$ of type i consumers, to be determined endogenously, will buy from firm A. The other $1 - \hat{x}_i^1$ will buy from firm B. We will refer to the consumers that buy from A in period 1 as segment A, and to the consumers that buy from B in period 1 as segment B. In **period 2**, firms A and B simultaneously each set 3 prices, observable to everyone. Firm A charges a **loyalty price** p_{AA}^2 to consumers that bought from A in period 1, a **poaching price** p_{AB}^2 to consumers that bought from B in period 1, and a **retention price** p_A^R to try to lure consumers that bought from firm A in period 1, but now contact that firm to cancel their

[1]Note that this applies in the telecom and car insurance examples in the introduction.

[2]In an earlier version, we did a different analysis. There we assume that prices are set sequentially in the second period; only after having observed the poaching price of their competitor do firms decide on their retention price. That would require the original supplier to be able to observe the poaching price from the other firm. In that case, our qualitative results still hold. Second-period prices in the model with the possibility of retention offers will be slightly higher – for the same reason that equilibrium prices in a differentiated-product Bertrand model with sequential moves are higher than equilibrium prices in a differentiated-product Bertrand model with simultaneous moves. Details are available from the authors upon request.

subscription. Similarly, B sets prices p^2_{BB}; p^2_{BA}; and p^R_B.

For the analysis that follows to be valid, we need to impose some parameter restrictions. In the benchmark model without retention offers, we require that for any λ some low type and some high type consumers are poached in the second period. As we will show in the analysis below, this requires;

$$z_L < 1; \tag{1}$$

$$z_H < 1/3 + 2z_L/3; \tag{2}$$

3 Benchmark: no retention offers

Preliminaries We first consider a benchmark without retention offers. In that case, only total switching costs z_i ar relevant. The timing of this simplified game is thus as follows. In **period 1**, A and B simultaneously set p^1_A and p^1_B, and a fraction $\hat{x}^1_i$ of type i consumers buys from A. These consumers comprise segment A, the others segment B. In **period 2**, A and B simultaneously set poaching prices and loyalty prices. We look for a symmetric equilibrium and solve with backward induction.

Second period In equilibrium some type i consumers in segment A will be tempted by the poaching price of firm B. The second period will then have some $\hat{x}^2_{Ai} < \hat{x}^1_i$ consumers again choosing for firm A, while the remaining $\hat{x}^1_i - \hat{x}^2_{Ai}$ switch to B. A similar analysis holds for segment B. The indifferent types i on segments A and B are given by

$$\hat{x}^2_{Ai} = \tfrac{1}{2}\left(1 + p^2_{BA} - p^2_{AA} + z_i\right); \qquad \hat{x}^2_{Bi} = \tfrac{1}{2}\left(1 + p^2_{BB} - p^2_{AB} - z_i\right), \tag{3}$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}^1_i$. Parameter restrictions (1) and (2) assure that that is the case in equilibrium.[3]

[3] To have $\hat{x}^2_{Ai} > 0$, we need $(2 + 3z_i - 2\bar{z}) > 0$, hence $2\bar{z} < 2 + 3z_i$. We want this to be satisfied for all λ. It is most restrictive for $\lambda = 0$, in which case it yields

$$2z_H < 2 + 3z_i. \tag{4}$$

For the high types, this is always satisfied. For the low types, it requires $2z_H - 3z_L < 2$.

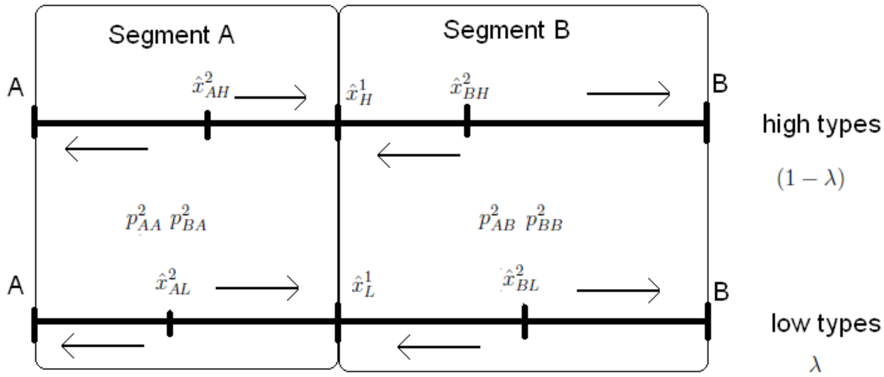


Figure 1: Market segmentation in both periods, Benchmark.

Figure 1 depicts this situation. The top panel reflects consumers with high switching costs, the bottom panel those with low switching costs. In period 1, those to the left of $\hat{x}_i^1$ buy from firm A.[4] Those to the right of $\hat{x}_i^1$ buy from B. In period 2, consumers in segment A that are located to the left of $\hat{x}^2_{AH}$ will buy from firm A, while those to the right will buy from B. Something similar applies to consumers in segment B. As $z_H > z_L$, we have from (3) that $x^2_{AH} > x^2_{AL}$ and $x^2_{BH} > x^2_{BL}$: as their switching costs are higher, fewer high types will switch in period 2.

Second-period profits for firm A are given by

$$\begin{aligned} \Pi^2_A &= \Pi^2_{AA} + \Pi^2_{AB} \\ &\equiv (p^2_{AA} - c)\left[\lambda \hat{x}^2_{AL} + (1-\lambda)\hat{x}^2_{AH}\right] \\ &\quad + (p^2_{AB} - c)\left[\lambda(\hat{x}^2_{BL} - \hat{x}^1_L) + (1-\lambda)(\hat{x}^2_{BH} - \hat{x}^1_H)\right], \end{aligned} \tag{5}$$

where Π^2_{AA} (the second line) reflects profits from loyal consumers, and Π^2_{AB}

In a symmetric equilibrium, we will have $\hat{x}_i^1 = 1/2$. For the second period, we thus need $\hat{x}^2_{Ai} < 1/2$, hence $2 + 3z_i - 2\bar{z} < 3$, so $3z_i - 2\bar{z} < 1$. We want this to be satisfied for all λ. It is most restrictive for $\lambda = 1$, in which case it yields $3z_i - 2z_L < 1$. For the low types, this requires (1). For the high types, it requires $3z_H - 2z_L < 1$, or (2). Note that if this is satisfied, (4) is satisfied as well.

[4] We will show below that $\hat{x}^1_H = \hat{x}^1_L$, as is depicted in the figure.

(the third line) those from consumers that are poached. Firm B's profits are

$$\begin{aligned}\Pi_B^2 &= \Pi_{BB}^2 + \Pi_{BA}^2 \\ &\equiv (p_{BA}^2 - c)\left[\lambda(\hat{x}_L^1 - \hat{x}_{AL}^2) + (1-\lambda)(\hat{x}_H^1 - \hat{x}_{AH}^2)\right] \\ &\quad +(p_{BB}^2 - c)\left[\lambda(1 - \hat{x}_{BL}^2) + (1-\lambda)(1 - \hat{x}_{BH}^2)\right]. \end{aligned} \tag{6}$$

For ease of exposition, we define $\bar{z}$ as the weighted average of switching costs in the population, and $\hat{x}^1$ as the weighted average location of indifferent consumers in period 1:

$$\bar{z} \equiv \lambda z_L + (1-\lambda) z_H, \tag{7}$$

$$\hat{x}^1 \equiv \lambda \hat{x}_L^1 + (1-\lambda)\hat{x}_H^1. \tag{8}$$

Using (3) we then have

$$\Pi_{AA}^2 = \frac{1}{2}(p_{AA}^2 - c)\left[1 + p_{BA}^2 - p_{AA}^2 + \bar{z}\right] \tag{9}$$

$$\Pi_{BA}^2 = (p_{BA}^2 - c)\left[\hat{x}^1 - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2 + \bar{z}\right)\right]. \tag{10}$$

Maximizing (9) with respect to p_{AA}^2 and (10) with respect to p_{BA}^2 yields the following reaction functions:

$$p_{AA}^2 = \tfrac{1}{2}\left(1 + p_{BA}^2 + c + \bar{z}\right); \qquad p_{BA}^2 = \tfrac{1}{2}\left(2\hat{x}^1 - 1 + p_{AA}^2 + c - \bar{z}\right). \tag{11}$$

Solving the system gives:

$$p_{AA}^2 = c + \tfrac{1}{3}(1 + 2\hat{x}^1 + \bar{z}); \qquad p_{BA}^2 = c + \tfrac{1}{3}(4\hat{x}^1 - 1 - \bar{z}). \tag{12}$$

We then immediately have

$$\hat{x}_{Ai}^2 = \frac{1}{6}\left(1 + 2\hat{x}^1 + 3z_i - 2\bar{z}\right) \tag{13}$$

and

$$\Pi_{AA}^2 = \tfrac{1}{18}(1 + 2\hat{x}^1 + \bar{z})^2; \qquad \Pi_{BA}^2 = \tfrac{1}{18}(4\hat{x}^1 - 1 - \bar{z})^2. \tag{14}$$

On segment B, we can do a similar analysis. Here

$$\begin{aligned}\Pi_{BB}^2 &= \frac{1}{2}(p_{BB}^2 - c)\left[1 + p_{AB}^2 - p_{BB}^2 + \bar{z}\right], \\ \Pi_{AB}^2 &= (p_{AB}^2 - c)\left[1 - \hat{x}^1 - \frac{1}{2}\left(1 + p_{AB}^2 - p_{BB}^2 + \bar{z}\right)\right].\end{aligned}$$

Hence

$$p_{AB}^2 = c + \tfrac{1}{3}(3 - 4\hat{x}^1 - \bar{z}); \qquad \Pi_{AB}^2 = \tfrac{1}{18}(3 - 4\hat{x}^1 - \bar{z})^2. \tag{15}$$

First period We now solve for the first period. Consumers are forward-looking and rationally take into account the events that will unfold in the second period. A consumer that is indifferent between A and B in period 1 thus anticipates that, whatever she chooses, she will switch in period 2. Denoting the discount factor by δ, the indifferent type i located at $\hat{x}_i^1$ has

$$\begin{aligned}r - \hat{x}_i^1 - p_A^1 &+ \delta(r - (1 - \hat{x}_i^1) - p_{BA}^2 - z_i) \\ &= r - (1 - \hat{x}_i^1) - p_B^1 + \delta(r - \hat{x}_i^1 - p_{AB}^2 - z_i),\end{aligned} \tag{16}$$

where the left-hand side gives her total lifetime utility if she chooses A in period 1, while the right-hand side gives that of choosing B in period 1. Note that switching costs z_i drop out of this equation; either way, in equilibrium this consumer will always incur switching costs in period 2, so these do not affect $\hat{x}_i^1$. This immediately implies $\hat{x}_L^1 = \hat{x}_H^1 = \hat{x}^1$. Solving (16) then gives

$$\hat{x}^1 = \frac{1 + p_B^1 - p_A^1 - \delta(1 + p_{BA}^2 - p_{AB}^2)}{2(1 - \delta)}. \tag{17}$$

Substituting second-period equilibrium prices from (12) and (15) and solving for $\hat{x}^1$ yields

$$\hat{x}^1 = \frac{1}{2} + \frac{3(p_B^1 - p_A^1)}{6 + 2\delta}. \tag{18}$$

In the first period, firm A sets p_A^1 as to maximize total discounted profits

$$\begin{aligned}\Pi_A &= (p_A^1 - c)\hat{x}^1 + \delta\Pi_{AA}^2 + \delta\Pi_{AB}^2 \\ &= (p_A^1 - c)\hat{x}^1 + \frac{\delta}{18}(1 + 2\hat{x}^1 + \bar{z})^2 + \frac{\delta}{18}(3 - 4\hat{x}^1 - \bar{z})^2. \end{aligned} \tag{19}$$

Taking the derivative with respect to p_A^1 :

$$\frac{\partial \Pi_A}{\partial p_A^1} = \hat{x}^1 + (p_A^1 - c)\frac{\partial \hat{x}^1}{\partial p_A^1} + \frac{2\delta}{9}\left(1 + 2\hat{x}^1 + \bar{z}\right)\frac{\partial \hat{x}^1}{\partial p_A^1} - \frac{4\delta}{9}\left(3 - 4\hat{x}^1 - \bar{z}\right)\frac{\partial \hat{x}^1}{\partial p_A^1}.$$

A symmetric equilibrium requires $p_A^1 = p_B^1$ hence $\hat{x} = \frac{1}{2}$. From (18), we have $\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-3}{6+2\delta}$. Hence, the first-order condition becomes

$$\frac{1}{2} - \frac{3}{6 + 2\delta}\left(p_A^1 - c + \frac{2\delta\bar{z}}{3}\right) = 0.$$

This yields equilibrium prices

$$p_A^1 = p_B^1 = c + 1 + \frac{\delta}{3}\left(1 - 2\bar{z}\right).$$

We thus have the following:

Theorem 1 *In the benchmark without retention offers, equilibrium first-period, loyalty and poaching prices are given by*

$$\begin{aligned} p_1^{bm} &= c + 1 + \frac{\delta}{3}\left(1 - 2\bar{z}\right); \\ p_{loyal}^{bm} &= c + \frac{1}{3}(2 + \bar{z}); \\ p_{poach}^{bm} &= c + \frac{1}{3}(1 - \bar{z}). \end{aligned} \tag{20}$$

Equilibrium profits are given by

$$\Pi^{bm} = \frac{1}{2} + \frac{1}{9}\delta\left(4 + \bar{z}^2 - 2\bar{z}\right). \tag{21}$$

Proof. In Appendix. ■

In a standard Hotelling model (or a model without switching costs), we would have $p = p^h \equiv c + 1$ in each period. From (1), we have $\bar{z} < 1$, hence $p^h > p^{bm}_{loyal} > p^{bm}_{poach}$. Hence, loyal consumers end up paying a higher price than those that are poached by the other firm ($p^{bm}_{loyal} > p^{bm}_{poach}$). Also, the possibility of poaching makes competition particularly fierce in the second period ($p^{bm}_{loyal} < p^h$). In the first period, the effect is ambiguous. On the one hand, consumers are less sensitive to first period prices;[5] marginal consumers know that if they are tempted to consume their less-preferred product, that will imply higher prices in period 2.[6] On the other hand, as switching costs increase, firms are more eager to attract consumers in period 1, as they will be less inclined to switch, so second-period profits increase.[7] As a result, first-period prices are higher ($p^{bm}_1 > p_h$) when average switching costs are low, but lower ($p^{bm}_1 < p_h$) when switching costs are high. We also have:

Corollary 1 *The total discounted price paid by both loyal and non-loyal consumers is lower than in a model without switching costs. All consumers are strictly better off. Firms are worse off. Total welfare decreases.*

Proof. In Appendix. ■

Hence, the effect of lower prices in the second period dominates a possible higher price in the first. As a result, consumers are worse off and firms are better off. The effect on welfare is immediate; prices are just a transfer but some consumers now no longer consume their favorite product in the second period and moreover have to incur switching costs.

4 Introducing retention offers

Preliminaries We now consider the full model and analyze whether there is an equilibrium in which retention offers occur. We thus look for an equi-

[5]Note from (18) that $\partial \hat{x}_1 / \partial \left(p^1_B - p^1_A\right) = 3/\left(6 + 2\delta\right)$, whereas in a standard Hotelling model, we would have $\partial \hat{x} / \partial \left(p_B - p_A\right) = 1/2$.

[6]From (12) an increase in $\hat{x}_1$ (the size of segment A) implies both p^2_{AA} and p^2_{BA} increase.

[7]From (19), in equilibrium $\partial \Pi_A / \partial \bar{z} = \delta \left(1 + 2\bar{z}\right)/9 > 0$.

librium where low types that do not switch always pay the retention price while high types that do not switch pay the loyalty price.

We solve with backward induction and again focus on segment A; consumers that have bought from firm A in period 1. For retention offers to occur in equilibrium, we need that low types that buy again from A go for the retention offer p_A^R, while high types prefer the loyalty price p_{AA}^2. For the low types, we thus need that the costs z_L^1 are smaller than the difference between p_A^R and p_{AA}^2, while for the high types that is not the case. An equilibrium with retention offers thus requires

$$z_L^1 < p_{AA}^2 - p_A^R < z_H^1. \tag{22}$$

Below we will derive the parameter restrictions such that these conditions are indeed satisfied.

Second period In the equilibrium we look for, all low types have already incurred the cancellation costs z_L^1. A low type that switches thus incurs an additional z_L^2 and would pay p_A^R when sticking to A. As in the benchmark, a high type that switches incurs an additional z_H and would pay p_{AA}^2 when sticking to A. Hence, the indifferent consumer in segment A is now given by

$$\hat{x}_{AL}^2 = \tfrac{1}{2}(1 + p_{BA}^2 - p_A^R + z_L^2); \qquad \hat{x}_{AH}^2 = \tfrac{1}{2}(1 + p_{BA}^2 - p_{AA}^2 + z_H). \tag{23}$$

Firm A's second-period profits from segment A now equal

$$\begin{aligned} \Pi_{AA}^2 &= \lambda(p_A^R - c)\hat{x}_{AL}^2 + (1-\lambda)(p_{AA}^2 - c)\hat{x}_{AH}^2 \\ &= \frac{1}{2}\lambda(p_A^R - c)\left(1 + p_{BA}^2 - p_A^R + z_L^2\right) \\ &\quad + \frac{1}{2}(1-\lambda)(p_{AA}^2 - c)\left(1 + p_{BA}^2 - p_{AA}^2 + z_H\right). \end{aligned} \tag{24}$$

Maximizing Π^2_{AA} with respect to p^R_A and p^2_{AA} yields, respectively,

$$p^R_A = \frac{1}{2}\left(1 + p^2_{BA} + z^2_L + c\right) \tag{25}$$
$$p^2_{AA} = \frac{1}{2}\left(1 + p^2_{BA} + z_H + c\right). \tag{26}$$

Denote by $\tilde{z}$ the weighted average of the additional switching costs that consumers incur relative to consumers that do not switch, and by $\tilde{p}_A$ the weighted average of these two prices:

$$\tilde{z} \equiv \lambda z^2_L + (1-\lambda)\, z_H \tag{27}$$
$$\tilde{p}_A \equiv \lambda p^2_{AA} + (1-\lambda)\, p^R_A. \tag{28}$$

We can use (25) and (26) to write a reaction function in terms of $\tilde{p}_A$:

$$\tilde{p}_A = \frac{1}{2}\left(1 + p^2_{BA} + c + \tilde{z}\right). \tag{29}$$

. Firm B's second-period profits on segment A are given by

$$\begin{aligned}\Pi^2_{BA} &= (p^2_{BA} - c)\left[\lambda\left(\hat{x}^1_L - \hat{x}^2_{AL}\right) + (1-\lambda)\left(\hat{x}^1_H - \hat{x}^2_{AH}\right)\right] \\ &= (p^2_{BA} - c)\left[\hat{x}_1 - \frac{1}{2}(1 + p^2_{BA} - \tilde{p}_A + \tilde{z})\right],\end{aligned} \tag{30}$$

using (28), the definition of $\tilde{p}_A$. Maximizing with respect to p^2_{BA} yields

$$p^2_{BA} = \frac{1}{2}(2\hat{x}_1 - 1 + \tilde{p}_A + c - \tilde{z}). \tag{31}$$

The system of reaction functions (29) and (31) is thus equivalent to the system (11) we derived for the benchmark, except that the relevant price for firm A is now the weighted average of the two prices it charges in segment A; and the relevant average switching costs is $\tilde{z}$ rather than $\bar{z}$.

Solving for equilibrium prices, we thus obtain

$$p^2_{BA} = c + \frac{1}{3}(4\hat{x}^1 - 1 - \tilde{z}), \tag{32}$$

and, using (25) and (26),

$$\begin{aligned} p_A^R &= c + \frac{1}{3}(1 + 2\hat{x}^1 + z_L^2) \\ p_{AA}^2 &= c + \frac{1}{3}(1 + 2\hat{x}^1 + z_H), \end{aligned} \tag{33}$$

while equilibrium market shares follow directly from (23):

$$\hat{x}_{AL}^2 = \tfrac{1}{12}\left(2 + 4\hat{x}^2 + 3z_L^2 - \tilde{z}\right); \qquad \hat{x}_{AH}^2 = \tfrac{1}{12}\left(2 + 4\hat{x}^2 + 3z_H - \tilde{z}\right), \tag{34}$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}_i^1$. Given that we already impose (1), parameter restriction (2) assures that is the case in equilibrium.[8] For equilibrium profits for A, we plug these values into (24) to find

$$\Pi_{AA}^2 = \frac{1}{8}\lambda\left(1 + z_L^2 + b\right)^2 + \frac{1}{8}\left(1 - \lambda\right)\left(1 + z_H + b\right)^2,$$

with

$$b \equiv \frac{4\hat{x}_1 - \tilde{z} - 1}{3}. \tag{35}$$

Similarly, using (30), profits for firm B can be shown to equal

$$\Pi_{BA}^2 = b\left[\hat{x}_1 - \frac{1}{4}\left(b + \tilde{z} + 1\right)\right)\right] = \frac{1}{4}b\left[4\hat{x} - \tilde{z} - 1 - b\right] = \frac{1}{2}b^2.$$

On segment B, we have a similar analysis that yields

$$p_{AB}^2 = c + a; \qquad \Pi_{AB}^2 = \tfrac{1}{2}a^2, \tag{36}$$

[8] It is immediate that $\hat{x}_{Ai}^2 > 0$. For $\hat{x}_{Ai}^2 < 1/2$, we need from (35) that $\max\left\{z_L^2, z_H\right\} + b < 1$. Note

$$b = \frac{1 - \tilde{z}}{3} = \frac{1 - \left(\lambda z_L^2 + (1 - \lambda)\, z_H\right)}{3}$$

which is increasing in λ. Sufficient for the condition to be satisfied for all λ is thus that it is satisfied for $\lambda = 1$, which is the case if $\max\left\{z_L^2, z_H\right\} + \frac{1}{3}\left(1 - z_L^2\right) < 1$. For the low types, this implies $z_L^2 < 1$, which is satisfied if (1) is. For the high types we need $z_H + \frac{1}{3}\left(1 - z_L^2\right) < 1$, hence $z_H < \frac{2}{3} + \frac{1}{3}z_L^2$, which is implied by (1) and (2).

with

$$a \equiv \frac{3 - 4\hat{x}^1 - \tilde{z}}{3}.$$

First period The indifferent consumer in period 1 is again given by (17): retention prices do not affect first-period market shares, as marginal consumers will always switch. Substituting for p^2_{AB} and p^2_{BA} from (32) and (36):

$$\hat{x}^1_i = \frac{1 + p^1_B - p^1_A - \delta\left(\frac{8\hat{x}^1 - 1}{3}\right)}{2(1-\delta)}.$$

Using (8), substituting from the above equations and solving for $\hat{x}^1$ yields

$$\hat{x}^1 = \frac{1}{2} + \frac{3\left(p^1_B - p^1_A\right)}{6 + 2\delta}.$$

Note that this is the same expression as (18). We now have

$$\begin{aligned} \Pi_A &= (p^1_A - c)\hat{x}^1 + \delta\Pi^2_{AA} + \delta\Pi^2_{AB} \qquad (37) \\ &= (p^1_A - c)\hat{x}^1 + \frac{\delta}{8}\lambda\left(b + 1 + z^2_L\right)^2 + \frac{\delta}{8}(1-\lambda)\left(b + 1 + z_H\right)^2 + \frac{\delta}{2}a^2. \end{aligned}$$

Taking the derivative with respect to p^1_A :

$$\begin{aligned} \frac{\partial \Pi_A}{\partial p^1_A} &= \hat{x}^1 + (p^1_A - c)\frac{\partial \hat{x}^1}{\partial p^1_A} + \frac{\delta\lambda}{4}\left(b + 1 + z^2_L\right)\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p^1_A} \\ &\quad + \frac{\delta}{4}(1-\lambda)\left(b + 1 + z_H\right)\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p^1_A} + \delta a\frac{\partial a}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p^1_A} \\ &= \hat{x}^1 + (p^1_A - c)\frac{\partial \hat{x}^1}{\partial p^1_A} + \frac{\delta}{4}\left(b + 1 + \tilde{z}\right)\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p^1_A} + \delta a\frac{\partial a}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p^1_A}. \end{aligned}$$

With

$$\begin{aligned} \frac{\partial b}{\partial \hat{x}^1} &= -\frac{\partial a}{\partial \hat{x}^1} = \frac{4}{3} \\ \frac{\partial \hat{x}^1}{\partial p^1_A} &= \frac{-3}{2\delta + 6}, \end{aligned}$$

the first-order condition becomes[9]

$$\hat{x}^1 - \frac{3(p_A - c) + \delta(b + z + 1 - 4a)}{2\delta + 6} = 0.$$

Equilibrium requires $p_A^1 = p_B^1$, hence $\hat{x} = 1/2$ and $a = b = \frac{1-\tilde{z}}{3}$. Solving for equilibrium prices then yields:[10]

Theorem 2 *With the possibility of retention offers, equilibrium first-period, loyalty, poaching and retention prices are given by*

$$\begin{aligned} p_1^{ret} &= c + 1 + \frac{1}{3}\delta(1 - 2\tilde{z}) \\ p_{loyal}^{ret} &= c + \frac{1}{2}(1 + b + z_H) \\ p_{poach}^{ret} &= c + b; \\ p_{retent}^{ret} &= c + \frac{1}{2}\left(1 + b + z_L^2\right), \end{aligned} \tag{38}$$

with $b = (1 - \tilde{z})/3$ and $\tilde{z}$ given by (27). Equilibrium profits are

$$\Pi_A^{ret} = \frac{1}{2}\left(p^1 - c\right) + \frac{\delta}{8}\lambda\left(b + 1 + z_L^2\right)^2 + \frac{\delta}{8}(1 - \lambda)(b + 1 + z_H)^2 + \frac{\delta}{2}b^2. \tag{39}$$

Comparing these results to those in a model without switching costs, we now have $p^h > p_{loyal}^{ret} > p_{retent}^{ret} > p_{poach}^{ret}$.[11] Again, loyal consumers end up paying a higher price than those that are poached by the other firm,

[9] It is readily checked that the second-order condition is satisfied as well.

[10] The proof that large deviations are not profitable in equilibrium goes along the same lines as in the benchmark case.

[11] Clearly $p_{loyal}^{ret} > p_{retent}^{ret}$. As $b = \frac{1-\tilde{z}}{3} < \frac{1}{2}$ we have that $p_{poach}^{ret} < p_{retent}^{ret}$. Note

$$\begin{aligned} p_{loyal}^{ret} - p_h &= \frac{1}{6}\left(1 - \left(\lambda z_L^2 + (1 - \lambda) z_H\right) + 3z_H - 3\right) \\ &= \frac{1}{6}\left((2 + \lambda) z_H - \lambda z_L^2 - 2\right) \end{aligned}$$

Using (2), the term in brackets is smaller than

$$(2 + \lambda)\left(\frac{1}{3} + \frac{2}{3}z_L^2\right) - \lambda z_L^2 - 2 = -\frac{1}{3}\left(1 - z_L^2\right)(4 - \lambda) < 0,$$

which establishes the result.

but the same applies to consumers that end up paying the retention price. Consumers that go for a retention offer thus pay a *higher* price than what they would pay if they would switch. As their original supplier knows that these consumers have a preference for their product, they do not have to fully compensate for the lower price of the other firm. The comparison of p_1^{ret} and p^h is again ambiguous, with similar comparative statics as in the benchmark case: only with low average switching costs, first-period prices are higher than in a world without switching costs.

Poaching prices are decreasing in switching costs z_H and z_L^2: the higher these, the more of an effort firms have to make to poach consumers. At the same time, an increase in these switching costs increases loyalty prices, as firms have to make less of an effort to retain consumers. These comparative statics are the same to those in the benchmark model. Retention prices increase in z_L^2. Only low types end up paying this price, and an increase in their switching costs makes it easier to retain them. First period prices decrease in z_H and z_L^2. As it becomes harder to poach consumers in the second period, it becomes more profitable to attract them in the first period. Hence, an increase in switching costs deceases first-period prices. This has the following effects:

Corollary 2 *The total discounted price paid by non-loyal consumers and by loyal low types, is lower than in model without switching costs. The total discounted price paid by loyal high types is lower for low enough λ and ambiguous otherwise. Firms are worse off. Total welfare decreases.*

Proof. In Appendix. ■

Anyhow, it is far more interesting to compare a world in which retention offers are possible to one where they are not; doing so allows us to truly evaluate the welfare effects of retention offers *per se*. We wil do so in the next section.

As a final technical aside, note that we need some parameter restrictions for our separating condition (22) to be satisfied. From (33), this requires

$$z_H^1 + z_H^2 > 2z_L^1 + z_L^2; \tag{40}$$

$$z_H^2 - z_H^1 < z_L^2. \tag{41}$$

5 The effects of retention offers

Comparing the model with the possibility of retention offers to the benchmark model, we now have the following:

Theorem 3 *Introducing the possibility of retention offers in our benchmark model has the following effects on prices:*

1. *First-period prices, poaching prices, and loyalty prices all increase.*
2. *New poaching prices are still lower than benchmark loyalty prices. New loyalty prices are higher than benchmark poaching prices.*
3. *Retention prices are higher than benchmark poaching prices, but lower than benchmark loyalty prices.*

Summarizing, we have $p_1^{ret} > p_1^{bm}$*, while the effects on second-period prices are as follows:*

	p_{poach}^{bm}	p_{loyal}^{bm}
p_{poach}^{ret}	$>$	$<$
p_{retent}^{ret}	$>$	$<$
p_{loyal}^{ret}	$>$	$>$

Proof. In Appendix. ■

To see what drives these results, note the following. First, for the low types, effective switching costs decrease. *Ceteris paribus*, when viewed in isolation, this would lead to a lower price charged by firm A and a higher price charged by firm B (see e.g. equation (12)). Second, the loyalty price will now only be paid by high types. These are reluctant to switch, leading to higher loyalty prices. In turn, when viewed in isolation, these higher loyalty prices also allow firm B to charge higher poaching prices. Both channels lead to higher poaching prices. Also, note that average effective switching costs decrease. That implies that firms become less eager to capture consumers in period 1, hence first-period prices increase.

The welfare effects of retention offers are as follows:

Theorem 4 *The possibility of retention offers increases equilibrium profits and decreases total consumer surplus. Each high type consumer is worse off. Only some individual low type consumers that end up paying the retention price, may be better off. Total welfare effects goes down.*

Proof. In Appendix. ■

Hence, although retention offers may at first sight seem to be good news for consumers, that is not the case when we take all equilibrium effects into account. In our model, retention offers serve to screen consumers with high switching costs from those with low switching costs, allowing firms to effectively price discriminate against high cost consumers, which hurts such consumers. Yet, consumers with low switching costs are often also worse off. They are forced to incur some of their switching costs in order to qualify for the retention offer, even if they do not intend to switch. Moreover, this lowers their effective switching costs, making competition for them less fierce in the first period. As a result, firms benefit from having the possibility of making retention offers.

6 Conclusion

In this paper, we studied the practice of retention offers. In a two-period Hotelling model, two firms practice behavior-based price discrimination. In the second period, they can try to poach consumers by offering them a better deal. However, firms can retaliate by making a retention offer. Consumers differ in their switching costs. In equilibrium, low-switching-cost consumers always solicit a retention offer, while this is too costly for high-switching-cost consumers to do so. As a result, retention offers allow firms to effectively price discriminate against high-switching-cost consumers.

We find that the possibility of retention offers increases firm profits. All high-cost consumers are worse off, but some low-cost consumers may benefit. Prices increase. From a welfare perspective, more wasteful switching costs are incurred, as all low-cost consumers solicit a costly offer from the competitor in order to secure a retention price.

Appendix: Proofs

Proof of Theorem 1

Most of the analysis is already contained in the main text. The second-period profit functions (5) and (6) are clearly concave – provided that firms set prices such that the indifferent high and low type consumers are both strictly between 0 and 1/2 in equilibrium. Yet, it may still be profitable to do a large defection. We will show that that is not the case. As in the main text, we focus on segment A.

First consider firm B. It can defect to a price p_{BA}^2 that is so high that it only sells to the low types. In equilibrium, that requires setting p_{BA}^2 such that $\hat{x}_{AH} \geq 1/2$, or $1 + p_{BA}^2 - p_{AA}^2 + z_H \geq 1$. This implies setting $p_{BA}^2 = p_{AA}^2 - z_H$. Its profits are then given by

$$\pi_{BL}^2 = \lambda \left(p_{BA}^2 - c\right) \left[\frac{1}{2} - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2 + z_L\right)\right],$$

which is maximized by setting $p_{BA}^2 = \frac{1}{2}\left(p_{AA}^2 + c - z_L\right)$. At $p_{BA}^2 = p_{AA}^2 - z_H$, these profits are decreasing whenever $p_{AA}^2 - z_H > \frac{1}{2}\left(p_{AA}^2 + c - z_L\right)$, hence if $z_H\left(\lambda + 5\right) - z_L\left(\lambda + 3\right) < 2$. This is most restrictive for $\lambda = 1$, so we need $z_H < \frac{1}{3} + \frac{2}{3}z_L$ which is exactly (2). Hence, we're on the downward sloping part of π_L. Therefore such a defection cannot be profitable.

Alternatively, firm B could set p_{BA}^2 so low that we serve all the low types, so $\hat{x}^2_{AL} = 0$. That implies setting

$$p^2_{BA} = p^2_{AA} - z_L - 1 = c + \frac{1}{3}(2 + \bar{z}) - z_L - 1 = c + \frac{1}{3}(\bar{z} - 1) - z_L < c,$$

which is clearly unprofitable.

Now consider firm A. First, it can defect by setting p^2_{AA} so high that it only sells to the high types. That requires setting p^2_{AA} such that $\hat{x}_{AL} \leq 0$ or $p^2_{AA} \geq 1 + p^2_{BA} + z_L$. Its profits are then given by

$$\pi^2_{AH} = \frac{1}{2}(1 - \lambda)\left(p^2_{AA} - c\right)\left[1 + p^2_{BA} - p^2_{AA} + z_H\right],$$

which is maximized by setting $p^2_{AA} = \frac{1}{2}\left(1 + p^2_{BA} + c + z_H\right)$. At $p^2_{AA} = 1 + p^2_{BA} + z_L$, these profits are decreasing whenever $1 + p^2_{BA} + c + z_H < 2p^2_{BA} + 2z_L + 2$ or $-2z_L - \frac{4}{3} + z_H + \bar{z}/3 < 0$, which is always the case. Hence such a defection cannot be profitable.

Finally, firm A can defect by setting a lower p^2_{AA}, such that it serves all the high types. In that case the profit function we use in the main text overestimates true profits (since it assumes a $\hat{x}_{AH} > 1/2$ rather than the true $\hat{x}_{AH} = 1/2$). As we cannot find a profit-increasing defection when looking at an inflated profit function, such a profit-increasing defection definitely does not exist when looking at the true profit function.

Proof of Corollary 1

For loyals, the effect on total discounted price is

$$\Delta P_{loyal} = p^{bm}_1 + \delta p^{bm}_{loyal} - (1 + c)(1 + \delta) = -\frac{1}{3}\bar{z}\delta < 0,$$

hence they are better off. For consumers that are poached

$$\Delta P_{poach} = p^{bm}_1 + \delta p^{bm}_{poach} - (1 + c)(1 + \delta) = -\frac{1}{3}\delta(1 + 3\bar{z}) < 0.$$

Poached consumers incur switching costs and a disutility from consuming their less preferred product in period 2. However, if they would choose not switch, they would still be better off than in a Hotelling model without switching costs, as prices are now lower. Revealed preference implies they are even better off when they do switch. For firms, lower prices and a covered market immediately imply that profits are lower. Total welfare is not affected by prices, but some consumers now incur switching costs, and a utility loss from consuming their less preferred product. Hence, welfare decreases.

Proof of Corollary 2

First consider the loyal consumers. Using (38),

$$\begin{aligned} P_{loyal}^{ret} &\equiv p_1^{ret} + \delta p_{loyal}^{ret} \\ &= 1 + c + \frac{1}{3}\delta(1 - 2\tilde{z}) + \delta\left(c + \frac{1}{2}(1 + b + z_H)\right) \end{aligned}$$

Hence, using (35),

$$\begin{aligned} P_{loyal}^{ret} - P^h &= P_{loyal}^{ret} - (1+\delta)(1+c) \\ &= \frac{1}{6}\delta(3z_H - 5\tilde{z}). \end{aligned}$$

With $\lambda = 0$, this equals $-\frac{1}{3}\delta z_H < 0$. With $\lambda = 1$, it is $\frac{1}{6}\delta\left(3z_H - 5z_L^1\right)$, which is ambiguous. For consumers that are poached, we have from (38)

$$\begin{aligned} P_{poach}^{ret} - P^h &\equiv p_1^{ret} + \delta p_{poach}^{ret} - (1+c)(1+\delta) \\ &= 1 + c + \frac{1}{3}\delta(1 - 2\tilde{z}) + \delta(c+b) - (1+c)(1+\delta) \\ &= -\frac{1}{3}\delta(1 + 3\tilde{z}) < 0. \end{aligned}$$

For consumers that pay the retention price, again using (27) and (38),

$$\begin{aligned} P_{retent}^{ret} - P^h &= p_1^{ret} + \delta p_{retent}^{ret} - (1+c)(1+\delta) \\ &= 1 + c + \frac{1}{3}\delta(1 - 2\tilde{z}) + \delta\left(c + \frac{1}{2}\left(1 + b + z_L^2\right)\right) - (1+c)(1+\delta) \\ &= \frac{1}{6}\delta\left(3z_L^2 - 5\tilde{z}\right) < 0 \end{aligned}$$

as $\tilde{z} \geq z_L^2$.

For profits, define $\hat{z}$ as the weighted average of the squares of additional switching costs: $\hat{z} \equiv \lambda \left(z_L^2\right)^2 + (1-\lambda)\left(z_H\right)^2$. From (39), we can then write

$$\Pi_A^{ret} = \frac{1}{2}\left(p_1^{ret} - c\right) + \frac{\delta}{8}\left((1+b)^2 + 2(1+b)\tilde{z} + \hat{z}\right) + \frac{\delta}{2}b^2.$$

Plugging in p_1^{ret}, using the fact that $\Pi^h = \frac{1}{2}(1+\delta)$ and rearranging, this implies

$$\Pi_A^{ret} - \Pi^h = \frac{\delta}{24}\left(6b(1+\tilde{z}) + 3\hat{z} - 2\tilde{z} + 15b^2 - 5\right).$$

Using the definition of b, this simplifies to

$$\Pi_A^{ret} - \Pi^h = \frac{\delta}{72}\left(9\hat{z} - \tilde{z}^2 - 16\tilde{z} - 4\right).$$

With $\hat{z} < \tilde{z}$, this implies $\Pi_A^{ret} < \Pi^h$. For welfare, the effect is immediate; some consumers no longer consume their preferred product in the second period, and all low type consumers now incur some switching costs.

Proof of Theorem 3

First note that, by construction, $z_H > \bar{z} > \tilde{z} > z_L^2$. We then have:

- $p_1^{ret} - p_1^{bm} = \frac{2\delta}{3}(\bar{z} - \tilde{z}) > 0$, hence $p_1^{ret} > p_1^{bm}$.
- $p_{poach}^{ret} - p_{poach}^{bm} = \frac{1}{3}(\bar{z} - \tilde{z}) > 0$, hence $p_{poach}^{ret} > p_{poach}^{bm}$.
- $p_{loyal}^{ret} - p_{loyal}^{bm} = \frac{1}{2}(1 + b + z_H) - \frac{1}{3}(2 + \bar{z}) = \frac{1}{6}(3z_H - \tilde{z} - 2\bar{z}) > 0$, hence $p_{loyal}^{ret} > p_{loyal}^{bm}$.
- $p_{poach}^{ret} - p_{loyal}^{bm} = \frac{1-\tilde{z}}{3} - \frac{1}{3}(2 + \bar{z}) < 0$, as the first term is strictly smaller than 1/3, while the second term is strictly bigger than 2/3. Hence $p_{poach}^{ret} < p_{loyal}^{bm}$.
- We established $p_{poach}^{ret} > p_{poach}^{bm}$. From section 4, $p_{retent}^{ret} > p_{poach}^{ret}$. Hence $p_{retent}^{ret} > p_{poach}^{bm}$.

- $p^{ret}_{retent} - p^{bm}_{loyal} = \frac{1}{2}\left(1 + z_L^2 + \frac{1-\tilde{z}}{3}\right) - \frac{1}{3}(2+\bar{z}) = \frac{1}{6}\left(3z_L^2 - \tilde{z} - 2\bar{z}\right) < 0$, hence $p^{ret}_{retent} < p^{bm}_{loyal}$.
- $p^{ret}_{loyal} - p^{bm}_{poach} = \frac{1}{2}\left(1 + z_H + \frac{1-\tilde{z}}{3}\right) - \frac{1}{3}(1-\bar{z}) > 0$,as the first term is strictly larger than 1/2, while the second is strictly smaller than 1/3. Hence $p^{ret}_{loyal} < p^{bm}_{poach}$.

This establishes the result.

Proof of Theorem 4

We proceed as follows. First, we compare the total discounted prices that consumers end up paying under different circumstances. These comparisons will prove useful in deriving our results. We then consider how individual consumers are affected, and look at total welfare. After that, we consider firm profits and total welfare, respectively.

The effect on total discounted prices

We can establish the following:

Lemma 1 *For a consumer that gets poached in the benchmark, introducing the possibility of retention offers increases the total discounted price paid. For a consumer that is loyal in the benchmark, the total discounted price increases if she is also loyal with retention offers and decreases if she then gets poached. If she ends up paying the retention price, her total discounted price decreases if δ or λ is sufficiently low.*

Summarizing, the effects are as follows:

	P^{bm}_{poach}	P^{bm}_{loyal}
P^{ret}_{poach}	$>$	$<$
P^{ret}_{retent}	$>$	$\gtrless$
P^{ret}_{loyal}	$>$	$>$

Proof. Results involving $P^{\text{bm}}_{\text{poach}}$ follow directly from Theorem 3, as does

$P^{\text{ret}}_{\text{loyal}} > P^{\text{bm}}_{\text{loyal}}$. Let us now consider the expression $\Delta P_{\text{rr-bl}} \equiv P^{\text{ret}}_{\text{retent}} - P^{\text{bm}}_{\text{loyal}}$.

$$\begin{aligned}\Delta P_{\text{rr-bl}} &\equiv P^{\text{ret}}_{\text{retent}} - P^{\text{bm}}_{\text{loyal}} = \frac{2\delta}{3}\left(\bar{z}-\tilde{z}\right) + \frac{1}{6}\left(3z_L^2 - \tilde{z} - 2\bar{z}\right) \\ &= \frac{1}{6}\left(-z_L^2\left(2\lambda + 4\lambda\delta - 3\right) - 4z_H\left(1-\lambda\right) - 2\lambda z_L\left(1-2\delta\right)\right)\end{aligned}$$

At $\lambda = 0$, we have that $p_1^{\text{ret}} = p_1^{\text{bm}}$, hence $\Delta P_{\text{rr-bl}} = P^{\text{ret}}_{\text{retent}} - P^{\text{loyal}}_{\text{bm}} < 0$. For $\lambda = 1$, we have

$$\Delta P_{\text{rr-bl}} = \left(\frac{2\delta - 1}{3}\right) z_L^1.$$

For $\delta < 1/2$ this is also negative. For $\delta > 1/2$ it is positive. Linearity in λ then implies that $\Delta P_{\text{rr-bl}}$ is always negative if $\delta < 1/2$, and is positive for λ sufficiently high if $\delta < 1/2$. Finally, consider the expression $\Delta P_{\text{rp-bl}} \equiv P^{\text{ret}}_{\text{poach}} - P^{\text{bm}}_{\text{loyal}}$. First note

$$\Delta P_{\text{rp-bl}} = \frac{2\delta}{3}\left(\bar{z}-\tilde{z}\right) + \frac{1-\tilde{z}}{3} - \frac{1}{3}(2+\bar{z})$$

For $\lambda = 0$, this yields

$$\Delta P_{\text{rp-bl}} = -\frac{1}{3}\left(1 + 2z_H\right) < 0$$

For $\lambda = 1$:

$$\begin{aligned}\Delta P_{\text{rp-bl}} &= \frac{2\delta}{3} z_L^1 + \frac{1-z_L^2}{3} - \frac{1}{3}(2+z_L) \\ &\leq \frac{1}{3}\left(z_L^1 - 2z_L^2 - 1\right) < 0,\end{aligned}$$

where the first inequality follows from $\delta \leq 1$, and the second from $z_L < 1$. As $\Delta P_{\text{rp-bl}}$ is linear in λ, this establishes the result. ■

The effect on consumer welfare

Lemma 2 *The possibility of retention offers makes all consumers strictly worse off, apart possibly from those that pay the retention price in period 2.*

Proof. For a single consumer, there are 6 possible options: she is poached both in the benchmark as well as in the scenario with retention offers; she is loyal in both cases, she is poached in the benchmark and loyal in the retention scenario; she is loyal in the benchmark but poached in the retention scenario; she is loyal in the benchmark, but pays a retention price in the retention scenario, or she is poached in the benchmark and pays a retention price in the retention scenario. We will refer to these 6 options as PP, LL, PL, LP, LR and PR respectively. Note that not all 6 options necessarily occur in equilibrium, depending on parameter values, either one may occur.

In all cases, the total discounted price that a consumer ends up paying is a disutility for that consumer. A consumer that is poached in the second period has an additional disutility of, first, the switching costs that she has to incur and, second, the utility mismatch that is caused by the fact that she does no longer consumer her preferred product. A consumer that pays the retention price in the second period has an additional disutility that consist of the additional costs she has to incur to prepare for a switch.

A consumer is worse off with the possibility of retention offers if the total disutility she ends up with then is higher than her total disutility in the benchmark. We will refer to the total disutility in scenario x if a consumer ends up paying a price of type y as D_y^x. Going through all possibilities:

PP The net difference in disutility in both scenarios equals that in total discounted price. As $P^{\text{ret}}_{\text{poach}} > P^{\text{bm}}_{\text{poach}}$, we thus have $D^{\text{bm}}_{\text{poach}} < D^{\text{ret}}_{\text{poach}}$.

LL The net difference in disutility equals that in total discounted prices. As $P^{\text{ret}}_{\text{loyal}} > P^{\text{bm}}_{\text{loyal}}$, we thus have $D^{\text{bm}}_{\text{loyal}} < D^{\text{ret}}_{\text{loyal}}$.

PL As this consumer chooses the poaching price in the benchmark, she has $D^{\text{bm}}_{\text{poach}} < D^{\text{bm}}_{\text{loyal}}$. With $D^{\text{bm}}_{\text{loyal}} < D^{\text{ret}}_{\text{loyal}}$, this implies $D^{\text{bm}}_{\text{poach}} < D^{\text{ret}}_{\text{loyal}}$.

LP As this consumer chooses the loyalty price in the benchmark, she has $D^{\text{bm}}_{\text{loyal}} < D^{\text{bm}}_{\text{poach}}$. With $D^{\text{bm}}_{\text{poach}} < D^{\text{ret}}_{\text{poach}}$, this implies $D^{\text{bm}}_{\text{loyal}} < D^{\text{ret}}_{\text{poach}}$.

LR In this case, consider $\Delta D_{\text{rr-bl}} \equiv D^{\text{ret}}_{\text{retent}} - D^{\text{bm}}_{\text{loyal}} = P^{\text{ret}}_{\text{retent}} + \delta z_L^1 - P^{\text{bm}}_{\text{loyal}}$. From the proof of Lemma 1, with $\lambda = 0$, we have that $P^{\text{ret}}_{\text{retent}} - P^{\text{bm}}_{\text{loyal}} =$

$\frac{1}{2}z_L^2 - \frac{2}{3}z_H < 0$, so

$$\Delta D_{\text{rr-bl}} = \frac{1}{2}z_L^2 - \frac{2}{3}z_H + \delta z_L^1 < \frac{1}{2}z_H - \frac{2}{3}z_H < 0.$$

With $\lambda = 1$, we have that $P_{\text{retent}}^{\text{ret}} - P_{\text{loyal}}^{\text{bm}} = \left(\frac{2\delta-1}{3}\right) z_L^1 > 0$, so $\Delta D_{\text{rr-bl}} > 0$, rendering the net effect ambiguous.

PR It is now important to take into account that the disutility of a consumer that is poached depends on her location, which we denote x. Consider $\Delta D_{\text{rr-bp}}(x) \equiv D_{\text{retent}}^{\text{ret}} - D_{\text{poach}}^{\text{bm}}(x)$ and denote by $m(x)$ the mismatch of a consumer located at $x \leq 1/2$ that gets poached. We then have

$$\begin{aligned}
\Delta D_{\text{rr-bp}}(x) &= P_{\text{retent}}^{\text{ret}} + \delta z_L^1 - P_{\text{poach}}^{\text{bm}} - \delta z_L - \delta m(x) \\
&= P_{\text{retent}}^{\text{ret}} - P_{\text{poach}}^{\text{bm}} - \delta z_L^2 - \delta m(x) \\
&= \frac{2\delta}{3}(\bar{z} - \tilde{z}) + \frac{\delta}{2}\left(1 + \frac{1-\tilde{z}}{3} + z_L^2\right) - \frac{\delta}{3}(1-\bar{z}) - \delta z_L^2 - \delta m(x)
\end{aligned}$$

Note that a poached consumer's transport costs are now $1-x$ where are x if she consumes her preferred product. Hence $m(x) = 1 - 2x$. Using the expressions for $p_{\text{loyal}}^{\text{bm}}$ and p_{retent}^{ret} in Theorems 1 and 2 we find for $\lambda = 0$ that $\Delta D_{\text{rr-bp}}(x) = \delta(12x - 4 + z_H - 3z_L^2)/6$, which is ambiguous. Similarly, for $\lambda = 1$, $\Delta D_{\text{rr-bp}}(x) = \delta\left(6x + 3z_L^1 - z_L^2 - 2\right)/3$ which is ambiguous as well.

■

Lemma 3 *For high types, total consumer welfare decreases with the possibility of retention offers. For low types, this is the case when λ is high enough.*

Proof. For the high types, this follows directly from Lemma 2 (note that high types never pay the retention price). The analysis for the low types is more involved. Consider segment A in the benchmark scenario. Total disutility of the low types that are loyal is given by $\hat{x}_{AL}^{\text{bm}} \cdot D_{\text{loyal}}^{\text{bm}} = \hat{x}_{AL}^{\text{bm}} \cdot P_{\text{loyal}}^{\text{bm}}$. Total disutility of the low types that are poached first consists of $\left(\frac{1}{2}-\hat{x}_{AL}^{\text{bm}}\right)\left(P_{\text{poach}}^{\text{bm}}+\delta z_L\right)$, as these consumers pay $P_{\text{poach}}^{\text{bm}}$ and also incur switching costs in the second period. Moreover, each of these consumers incurs a mismatch: her transportation costs are now $1-x$ whereas they would have been x if she consumed her preferred product. Hence $m(x)=1-2x$, and the total size of this mismatch equals

$$M_{AL}^{\text{bm}}=\int_{\hat{x}_{AL}^{\text{bm}}}^{1/2} m(x)dx=\left(\frac{1}{2}-\hat{x}_{AL}^{\text{bm}}\right)^2.$$

Hence total disutility of the low types in the benchmark is given by

$$D_L^{\text{bm}}\equiv 2\hat{x}_{AL}^{\text{bm}}\cdot P_{\text{loyal}}^{\text{bm}}+2\left(\frac{1}{2}-\hat{x}_{AL}^{\text{bm}}\right)\left(P_{\text{poach}}^{\text{bm}}+\delta z_L\right)+2\left(\frac{1}{2}-\hat{x}_{AL}^{\text{bm}}\right)^2.$$

With the possibility of retention offers low type consumers can obtain a retention offer against cost z_L^1. Therefore total disutility of low types with the possibility of retention offers is given by

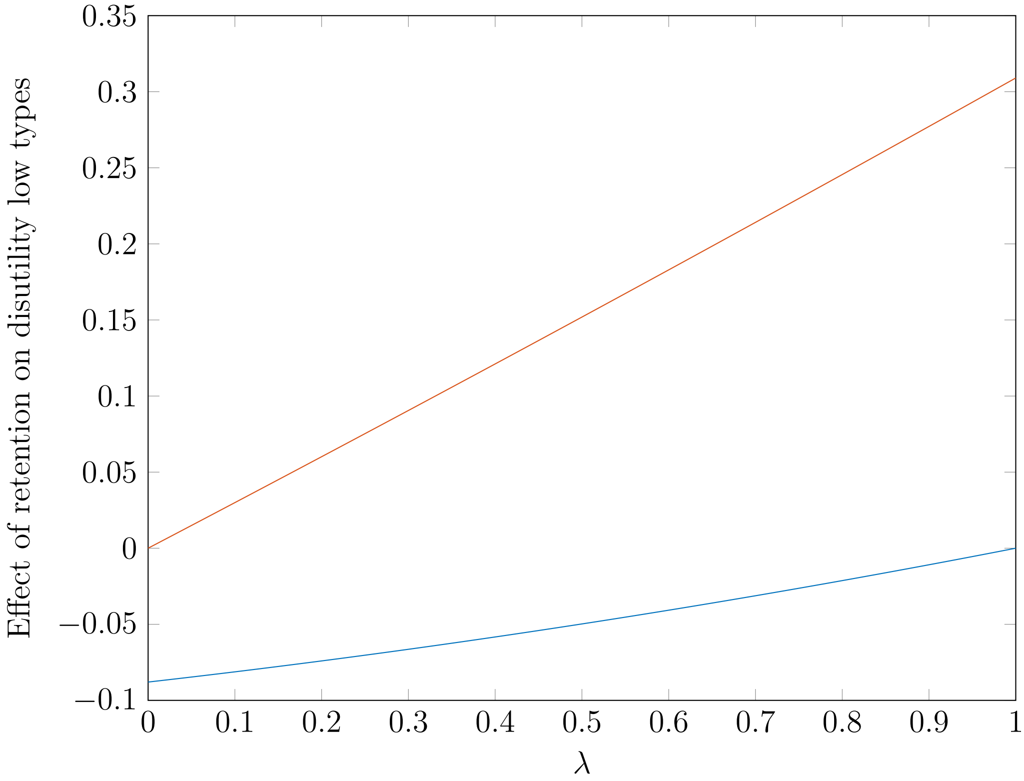
$$D_L^{\text{ret}}\equiv 2\hat{x}_{AL}^{\text{ret}}\cdot\left(P_{\text{retent}}^{\text{ret}}+z_L^1\right)+2\left(\frac{1}{2}-\hat{x}_{AL}^{\text{ret}}\right)\left(P_{\text{poach}}^{\text{ret}}+\delta z_L\right)+2\left(\frac{1}{2}-\hat{x}_{AL}^{\text{ret}}\right)^2.$$

It turns out to be impossible to compare these two expressions analytically. We therefore resort to a numerical analysis. For all values of λ, Figure 2 gives for all admissible values of z_L^1, z_L^2, z_H^1, and z_H^2. the upper and the lower bound on the net welfare effects of the low types of the possibility of having retention offers (thus on $D_L^{\text{ret}}-D_L^{\text{bm}}$ as defined above), [12]

From the figure, we have that welfare of the low types may improve with retention offers for low enough λ. Only for those λ, we saw that the low types that buy from A in both scenarios do pay a lower price under retention, while the number of low types that gets poached decreases, lowering their costs of

[12] The analysis was done in MATLAB. For each of 100 values of λ between 0 and 1, we consider 50 values of z_L^1, z_L^2, z_H^1, as well as z_H^2 to find the highest and the lowest possible value of the effect, taking into account the parameter conditions that have to be satisfied (thus: $z_L^1<z_H^1$; $z_L^2<z_H^2$, and conditions (1), (2), (40) and (41)). The MATLAB code is available upon request. Looking at a finer grid did not appreciably affect the outcomes. In all figures, we take $\delta=1$: the size of δ does not affect the qualitative analysis as all comparisons are proportional to δ.

Figure 2: Effect on disutility of low types of the possibility of retention offers.



The figure gives for all feasible values of switching costs the upper and lower bound of the effect of the possibility of retention offers on total disutility of the low types, as a function of lambda.

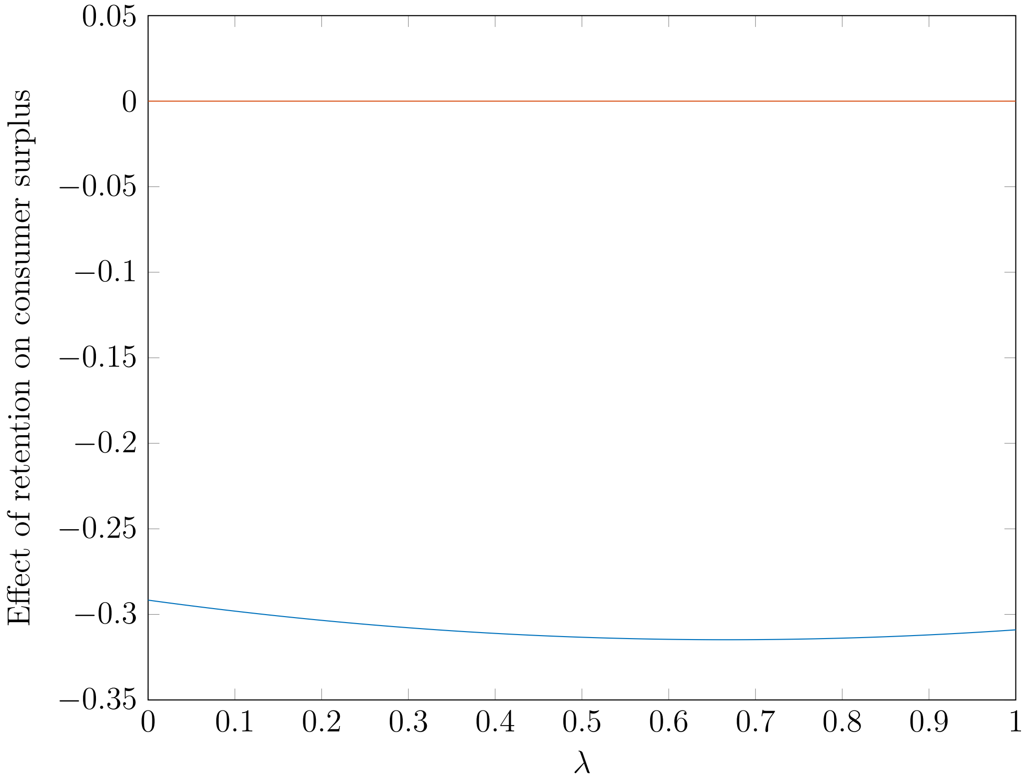
mismatch. For low λ, these positive effects outweigh the negative effects of a higher poaching price and costs to secure a competing offer.

Figure 3 shows the change in total consumer welfare. From the figure, for low λ, the positive effects on the low types is outweighed by the negative effects on the high types, rendering the net effect negative. For higher values, the effect is negative for both. ■

The effect on profits

First note that equilibrium profits would obviously increase if all total discounted prices in the retention scenario would be higher than those in the benchmark. Unfortunately, that is not the case. From Lemma 1, consumers

Figure 3: Effect on consumer surplus of the possibility of retention offers.



The figure gives for all feasible values of switching costs the upper and lower bound of the effect of the possibility of retention offers on total consumer surplus, as a function of lambda.

may end up paying a lower price if they are loyal in the benchmark, but are either poached or pay the retention price in the case where retention offers are possible.

Note that, in equilibrium, from (37) and Theorem 2, profits with the possibility of retention equal

$$\Pi_A^{ret} = \frac{1}{2}\left(1 + \frac{1}{3}\delta\left(1 - 2\tilde{z}\right)\right) + \frac{\delta}{8}\lambda\left(b + 1 + z_L^2\right)^2 + \frac{\delta}{8}\left(1 - \lambda\right)\left(b + 1 + z_H\right)^2 + \frac{\delta}{2}b^2.$$

From (19) and Theorem 1, profits in the benchmark can be written

$$\Pi^{bm} = \frac{1}{2}\left(1+\frac{1}{3}\delta\left(1-2\bar{z}\right)\right)+\frac{\delta}{8}\left(\frac{1-\bar{z}}{3}+1+\bar{z}\right)^2+\frac{\delta}{2}\left(\frac{1-\bar{z}}{3}\right)^2$$

Concavity of x^2 then implies

$$\begin{aligned}\Pi^{bm} &< \frac{1}{2}\left(1+\frac{1}{3}\delta\left(1-2\bar{z}\right)\right)+\frac{\delta}{8}\lambda\left(\frac{1-\bar{z}}{3}+1+z_L\right)^2 \\ &\quad +\frac{\delta}{8}\left(1-\lambda\right)\left(\frac{1-\bar{z}}{3}+1+z_H\right)^2+\frac{\delta}{2}\left(\frac{1-\bar{z}}{3}\right)^2 \end{aligned} \tag{42}$$

Consider the function

$$\begin{aligned}f(x) &\equiv \frac{1}{2}\left(1+\frac{1}{3}\delta\left(1-g\left(x\right)\right)\right)+\frac{\delta}{8}\lambda\left(\frac{1-g(x)}{3}+1+x\right)^2 \\ &\quad +\frac{\delta}{8}\left(1-\lambda\right)\left(\frac{1-g(x)}{3}+1+z_H\right)^2+\frac{\delta}{2}\left(\frac{1-g(x)}{3}\right)^2,\end{aligned}$$

with $g(x)\equiv\lambda x+\left(1-\lambda\right)z_H$. Note that

$$\frac{\partial f(x)}{\partial x}=-\frac{\delta}{12}\lambda\left(\frac{1}{3}\left(z_H-x\right)\left(1-\lambda\right)+\frac{8}{3}\left(1-x\right)\right).$$

This implies that the RHS of (42), which can be written $f(z_L)$, is smaller than $\Pi_A^{ret}=f\left(z_L^2\right)$, as $z_L^2<z_L$. This implies that $\Pi^{bm}<\Pi^{ret}$.

The effect on total welfare

Figure 4 reports on an analysis that is very similar to that in Figures 3 and 4, but now for total welfare.

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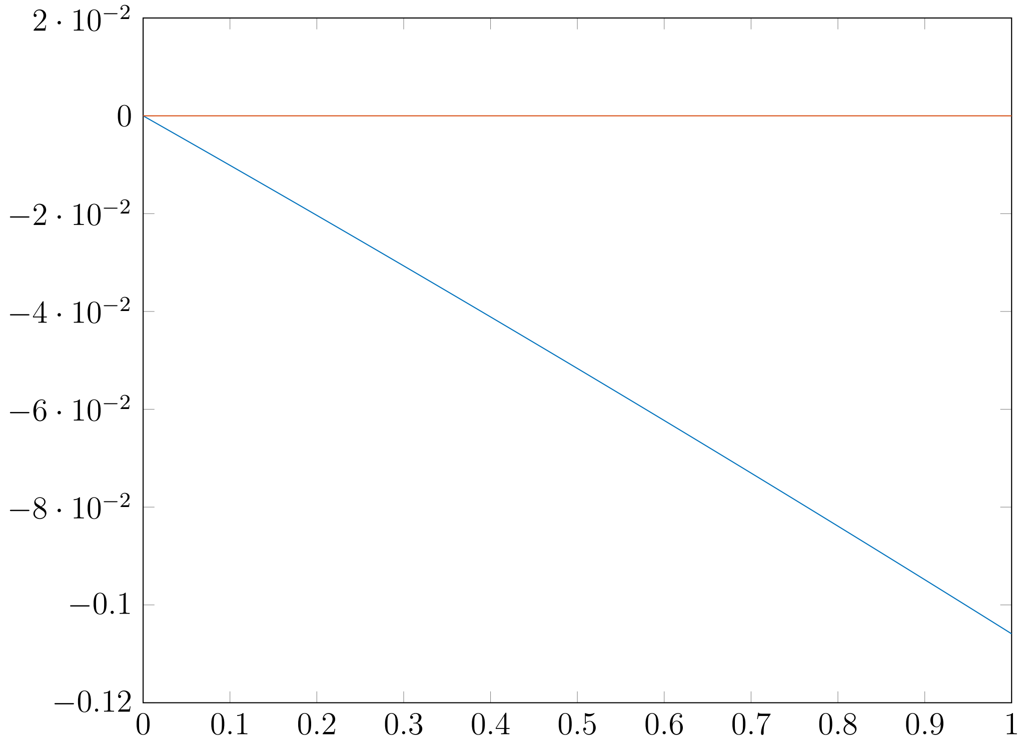
Figure 4: Effect on welfare of the possibility of making retention offers.



The figure gives for all feasible values of switching costs the upper and lower bound of the effect of the possibility of retention offers on total welfare, as a function of lambda.

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