# Competition with List Prices

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**Abstract**

Retail prices in stores are often lower than widely advertised list prices. We study the competitive role of such list prices in a homogeneous product duopoly where firms first set list prices before setting possibly reduced retail prices. Building on Varian (1980), we assume that some consumers observe no prices, some observe all prices, and some only observe the more salient list prices. We show that when the latter group chooses myopically, firms' ability to use list prices lowers average transaction prices. This effect is weakened when these consumers are rational. The possibility to use list prices facilitates collusion.

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# 1 Introduction

In many markets, in-store prices are frequently lower than the prices that are widely advertised. For example, electronics, fashion, or furniture retailers often advertise prices on television, radio, in printed catalogs, or via sponsored Internet ads, but then offer further discounts in-store and/or on their websites.[1] Similarly, manufacturers often quote a list price or suggested retail price in their ads or on their websites, but it is hardly a secret that the actual price consumers will have to pay is usually lower. In the Dutch retail gasoline market, majors operate numerous outlets that all charge different prices, but use a recommended retail price that is widely publicized.[2] Consumers know that they will never face a retail price that is higher than the recommended retail price of the brand they visit. Often, the price will be significantly lower. For ease of exposition, we will refer to the originally advertised/quoted prices as list prices in the remainder of this paper.[3]

Arguably, if such list prices are less transient and more visible than the actual retail prices set, some consumers may base their purchase decisions solely or primarily on them. Hence, retailers may be able to strategically use list prices to steer some consumer groups towards them, even though what ultimately matters to consumers is the actual retail price they will face. On the other hand, publicizing a low list price restricts a firm's pricing flexibility and may provoke aggressive discount competition by its rivals. This is particularly true since, by using price comparison websites, mobile phone apps, etc., in modern marketplaces there will typically also be a group of consumers that

[1] Examples of such further discounts are daily promotions and clearance sales.

[2] See e.g. `http://www.nu.nl/brandstof`.

[3] Indeed, Merriam-Webster defines a list price as "the basic price of an item as published in a catalog, price list, or advertisement before any discounts are taken" (see `https://www.merriam-webster.com/dictionary/list%20price`).

*is* well informed about the current, actual retail prices.[4] It is precisely the implications of these aspects that we explore in this paper.

In our model, two firms sell a homogeneous product and compete in prices in a two-stage game. In the first stage, they set list prices. In the second stage, after having observed each other's list price, they set retail prices. We build on the seminal Varian (1980) framework, where consumers are either informed and buy from the cheapest firm, or are uninformed and pick a firm at random. We introduce a third type: *partially informed* consumers that are uninformed about retail prices, but *are* informed about list prices, simply because these are more prominent.

Crucially, we assume that list prices are an upper bound on the retail prices that can be set. There can be many reasons for this. Firms may fear reputational losses when surprising consumers with a retail price that exceeds their list price, resulting in a decrease in future sales. Consumers may outright reject such a retail price due to loss aversion, anger, or other behavioral reasons, rendering the practice unprofitable.[5] Also, many countries simply have laws that prohibit such misleading advertising.[6]

In our model, we study how the use of list prices affects product market competition. In a competitive context, does the possible use of list prices benefit firms? Do higher list prices imply higher retail prices? How frequent and how deep are the discounts granted off list prices? Are consumers better off

[4]Coming back to the example of the Dutch retail gasoline market, this group may consist of consumers using popular mobile apps for gasoline price comparison such as "DirectLease Tankservice" and "ANWB Onderweg". Other consumers may be less well informed and just take into account the recommended retail prices publicized by the different brands (which are, next to the aforementioned website, also prominently displayed at gasoline stations), still others may just buy at a random station when they run out of fuel.

[5]See Bruttel (2018) for experimental evidence that demand tends to drop sharply for prices that exceed a recommended price, even if the latter has no informational content.

[6]See e.g. Rhodes and Wilson (2018) for a discussion of false-advertising regulations in the US and the European Union.

as they (or at least some of them) become better informed, or more sophisticated? Also, do list prices facilitate collusion? Does collusion in list prices raise retail prices and, if so, how?

In our baseline analysis, we assume that partially informed consumers are myopic and simply go to the firm with the lower list price. We then have a mixed-strategy equilibrium in the list-price stage, often followed by a mixed-strategy equilibrium in the retail-price stage. It is hard to explicitly characterize the equilibrium distribution of list prices: this involves solving a functional differential equation, where the solutions in different intervals stem from interdependent differential equations. For part of the parameter space, we can provide a semi-analytic solution. For all other cases, the equilibrium can be approximated using a simple numerical method.

With myopic consumers, we find the following. Firms always use list prices that effectively constrain their retail prices. The firm with the higher list price offers more frequent and deeper discounts.[7] With list prices sufficiently close to each other, this firm will even set a lower retail price on average. On aggregate, the use of list prices decreases expected profits and increases consumer surplus. Firms face a prisoners' dilemma: each has an incentive to use list prices to try to attract the partially informed, yet when both do, expected profits are lower.

We often find search externalities: having better informed consumers leads to lower average prices for all. This is the case when uninformed consumers become either partially or fully informed. When partially informed become fully informed, the effect is however ambiguous. For a given share of uninformed consumers, firms prefer a balanced mix of fully and partially informed consumers; harsh competition for either group is unfavorable.

[7] The only exception is when firms' list prices are so far apart that the firm with the higher list price has no incentive anymore to compete for informed consumers.

Solving for the case of rational consumers introduces further complexity. Note that in some subgames, the pricing equilibrium derived for the myopic case has partially informed consumers buying from the firm with the higher expected retail price. With rational consumers, the subgame equilibrium in such cases requires that partially informed consumers distribute themselves across firms such that their expected prices are equalized. Undercutting the competitor's list price thus no longer attracts all partially informed consumers, which reduces the incentive to do so. As a result, if the number of informed consumers is sufficiently large, firms no longer use effective list prices. Otherwise, we again have an equilibrium in mixed strategies.[8] Also in this case, we have to solve numerically. But this becomes more difficult as the equilibrium list-price distribution may involve multiple mass points and gaps.

Compared to the myopic case, average transactions prices are now higher. Firms thus benefit from facing rational rather than myopic consumers. Competition is less fierce in the list-price stage, which in turn relaxes it in the retail-price stage. In the terminology of Armstrong (2015), we thus have a *ripoff externality* when consumers become more strategically savvy and better understand the game being played.[9]

We also investigate how the ability to use list prices affects collusion. Successful collusion in list prices also increases average retail prices in our model. We thus provide a novel theory of harm for list-price collusion.[10] We also show that list prices facilitate collusion in a world with myopic consumers and grim-

[8]Technically, the lack of a pure-strategy equilibrium is no longer caused by the profit function being discontinuous, but rather by it failing to be quasi-concave.

[9]Unfortunately, further comparative statics results for the rational case are difficult to obtain, as the instability of the mixed-strategy equilibria make precise numerical approximations infeasible with the available methods.

[10]In competition law, a theory of harm is a theoretical underpinning of why firm behavior restricts competition and thereby lowers (consumer) welfare.

trigger strategies. In a nutshell, the possibility to use list prices does not affect the perfectly collusive outcome, but does lower punishment profits. Defection profits may be higher, but this does not outweigh the lower punishment profits.

As noted, we study a two-stage game with interlinked price competition, where firms often mix in both stages. To our knowledge, the only other model with that feature is Obradovits (2014), which studies competition under a specific intertemporal price regulation. Another feature of our model is that, with rational consumers, their strategic behavior may involve mixing which firm to visit. In Janssen et al. (2005), uninformed consumers also mix, but only in whether to enter the market, not in which firm to visit. As in our model list prices serve to steer the partially uninformed consumers, our work is also connected to models of price-directed consumer search, see e.g. Haan et al. (2018), Choi et al. (2018), and, in particular, Ding and Zhang (2018).

Our paper fits a small literature on list prices that serve as an upper bound on retail prices. Myatt and Ronayne (2019) also consider a two-stage modification of Varian (1980) where firms first set binding list prices and then retail prices. They do not have partially informed consumers, and focus on asymmetric pure-strategy equilibria with stable price dispersion. In equilibrium, firms never use discounts off list prices. In Díaz et al. (2009), list prices also enable pure-strategy equilibria where these otherwise do not exist, but in the context of capacity constraints. Committing to a low list price relaxes competition in the discounting stage. The use of list prices then increases profits.

Gill and Thanassoulis (2016) study a Hotelling model with price takers (that always buy at list prices) and bargainers (that obtain an endogenously determined discount with some probability). The ability to give discounts increases profits and reduces consumer surplus. In Anderson et al. (2019), firms offer personalized discounts from posted list prices. In equilibrium, 'captive consumers' (who strongly prefer some product) buy at the list price, while 'contested consumers' receive poaching and retention offers. The discounting stage yields a mixed-strategy equilibrium, but there is a pure-strategy equilibrium in list prices. The effect on prices and profits is ambiguous.

In Rao (1991), a national brand and a private label first set list prices, then choose the depth of discounts, and finally their frequency. In Chen and Rosenthal (1996a,b), firms use a binding list price as a commitment to convince potential buyers to further inspect their product. In Banks and Moorthy (1999), firms use list and promotional prices to price discriminate between consumers with high and low search costs.

Other papers consider list or recommended retail prices that are non-binding. Some focus on vertical relations. Buehler and Gärtner (2013) argue that manufacturers are better informed about demand and use recommended prices to convey this information to retailers. In Lubensky (2017) it is consumers that are better informed about market conditions. In Harrington and Ye (2019), intermediate goods producers may collude on high list prices to signal high costs to prospective buyers, hence affecting bargaining. Boshoff et al. (2018) note that non-binding price announcements can increase collusive profits by reducing asymmetric information between firms. Other such theories are discussed in Boshoff and Paha (2021); see also Andreu et al. (2020).

Our paper is also related to the behavioral industrial organization literature, where firms try to exploit boundedly rational consumers. In Puppe and Rosenkranz (2011), firms may benefit from recommended prices if consumers are loss averse. In Heidhues and Kőszegi (2014), high list prices set for an extended period serve as a reference price. This boosts demand during sales, and hence can increase profits. Paha (2019) studies list price collusion when the willingness-to-pay of loss-averse consumers is anchored to list prices.

Lastly, our model shares characteristics with the literature on competitive couponing (Shaffer and Zhang, 1995; Bester and Petrakis, 1996), where firms set regular prices, but can additionally send out coupons that grant discounted prices. In our model, such price discrimination is not feasible.

The remainder of this paper is organized as follows. In Section 2, we introduce the model. Section 3 analyzes the game with myopic partially informed consumers. In Section 4, we explore the case where partially informed consumers are rational. Section 5 examines the effects of, and scope for collusion. We conclude in Section 6. All proofs are relegated to Appendix A. In Appendix B, we outline our numerical procedure to approximate the equilibrium choice of list prices for the case of myopic partially informed consumers.

## 2 The game

We consider a market with two risk-neutral, profit-maximizing firms $i = 1, 2$ that sell a homogeneous good and compete in prices. Their marginal costs of production are normalized to zero. A unit mass of consumers have unit demand and a common willingness to pay that is normalized to one. The following events unfold. First, each firm simultaneously and unilaterally chooses its list price $P_i$. Second, after having observed all list prices, each firm decides on the retail price $p_i$ that it charges in its store. Reflecting the discussion in the introduction, we impose that a firm's retail price cannot exceed its list price, so $p_i \leq P_i$.[11] Third, consumers make purchase decisions.

[11] Loss aversion is one reason why it may be unattractive for firms to set prices above the list price. Suppose the list price is a reference point. If the retail price exceeds the list price, then the consumer experiences a loss when purchasing the product. For sufficiently high levels of loss aversion, consumers may simply not purchase the product anymore. Such a severe reaction would make it unprofitable for firms to exceed the list price. For this argument to work, we need to assume that the uninformed consumers become aware of the list price of the firm where they intend to purchase.

There are three types of consumers. A fraction $1 - \lambda - \mu$ is uninformed. They pick a firm at random and buy there, provided that the retail price does not exceed their willingness to pay. A fraction $\lambda$ is fully informed. These consumers observe all retail prices and buy from the cheapest firm. Hence, these two consumer types correspond to the uninformed and informed consumers in the classic Varian (1980) model. But we also assume that a fraction $\mu$ of consumers is *partially informed*. These consumers only observe list prices, pick a firm based on that information and buy there, again provided that the retail price does not exceed their willingness to pay. Throughout, we assume that all consumer types have strictly positive measure, so $\lambda > 0$, $\mu > 0$ and $\lambda + \mu < 1$.

We study two scenarios. First, in Section 3, we assume that the partially informed consumers use a simple rule of thumb and go to the firm with the lowest list price. As it turns out, this is however not always the optimal thing to do: in some pricing subgames, the equilibrium then has the firm with the lower list price charging a higher retail price on average. We therefore refer to the partially informed as being myopic in this scenario. In Section 4, we modify the analysis by assuming that the partially informed are rational, and hence do not visit a firm with a higher expected retail price.

# 3 Myopic partially informed consumers

In this section, we consider the case that partially informed consumers are myopic and buy from the firm with the lowest list price. We solve using backward induction. In Section 3.1, we characterize the equilibrium of all possible retail pricing subgames (stage 2). Then, in Section 3.2, we solve for the equilibrium in list prices (stage 1). Welfare implications are discussed in Section 3.3.

## 3.1 Equilibrium in the pricing subgames

First, for any two list prices set in stage 1, we derive the equilibrium in stage 2. As the analysis is fairly standard, we restrict attention to the main arguments and relegate the details to Appendix A.

**Preliminaries.** In case of different list prices, we refer to the firm with the lower list price as $L$, the other as $H$. Their respective list prices are denoted by $P_L$ and $P_H$. Firm $H$ will surely attract its share of uninformed consumers. Its mass of 'captive' consumers is thus given by

$$\alpha_H \equiv \frac{1-\lambda-\mu}{2}. \tag{1}$$

Firm $L$ will also attract the $\mu$ partially informed for sure. Hence, its mass of captive consumers is

$$\alpha_L \equiv \frac{1-\lambda-\mu}{2} + \mu = \frac{1-\lambda+\mu}{2}. \tag{2}$$

The remaining $\lambda = 1 - \alpha_H - \alpha_L$ consumers buy from the cheapest firm.

Define the ratio of list prices as $R$, i.e.,

$$R \equiv \frac{P_H}{P_L}. \tag{3}$$

By construction, $R > 1$. In case of equal list prices, we let $R = 1$; in this case, we assume that the partially informed choose randomly which firm to visit.

**Equilibrium characterization.** First, if $P_L$ is much smaller than $P_H$ (so $R$ is large), firm $L$ will simply set $p_L = P_L$, while firm $H$ will charge $p_H = P_H$. For this to be an equilibrium, undercutting $p_L = P_L$ should not be worthwhile for $H$ even though it attracts all informed consumers. This requires $(\alpha_H + \lambda)P_L \leq$

$\alpha_H P_H$, so $R \geq \frac{1-\alpha_L}{\alpha_H}$. If the two list prices are closer to each other, undercutting $P_L$ *is* profitable for $H$, and an equilibrium in pure strategies fails to exist.

Now suppose $P_1 = P_2 = P$, so $R = 1$. The subgame then collapses to Varian (1980) with $\lambda$ informed and $1-\lambda$ uninformed consumers, and an upper bound on prices $P$. In equilibrium, both firms draw their price from some cumulative distribution function (CDF) $F(p)$ on $\left[\underline{p}, P\right]$. Firm 1's expected profit from charging any $p \in \left[\underline{p}, P\right]$ is

$$\pi(p) = \left(\frac{1-\lambda}{2} + \lambda\left(1 - F(p)\right)\right)p,$$

as it sells to its share $\frac{1-\lambda}{2}$ of uninformed consumers for sure, and to the mass $\lambda$ informed consumers if it charges a price lower than its rival. In equilibrium, all $p \in \left[\underline{p}, P\right]$ should yield the same expected profit, $\pi(p) = \pi(P) = \frac{1-\lambda}{2}P$. Solving for $F(p)$ then gives

$$F(p) = \frac{1+\lambda-(1-\lambda)P/p}{2\lambda},$$

with support $\left[\frac{1-\lambda}{1+\lambda}P, P\right]$.

With $R$ sufficiently close but not equal to 1, the equilibrium is similar to that in Narasimhan (1988). That paper has (in our notation) $\alpha_L > \alpha_H$, but $P_L = P_H \equiv P$. Its equilibrium has both firms mixing on some $[\underline{p}, P]$ but in addition firm $L$ (with more captive consumers) has a mass point at $P$. The mass point assures that both firms are willing to mix on the exact same interval, which must necessarily be the case in equilibrium.[12] For $R$ close to 1, our equilibrium is qualitatively the same: both firms mix on some $[\underline{p}, P_L]$, and in addition, firm $L$ has a mass point at $P_L$.

For somewhat larger $R$, the above equilibrium breaks down, as $H$ would then rather deviate to $p_H = P_H$. For such $R$, the equilibrium is similar to the

[12]It makes no sense for one firm to mix among prices on which the other firm puts zero probability mass.

subgame equilibrium in Obradovits (2014). The second stage in that paper has (in our notation) $P_H > P_L$, but $\alpha_L = \alpha_H$. Its equilibrium has both firms mixing on some $[\underline{p}, P_L]$, but firm $L$ has a mass point at $P_L$, while firm $H$ has one at $P_H$. The probability masses assure that both firms are willing to mix on the same interval $[\underline{p}, P_L]$. For intermediate $R$, our equilibrium is qualitatively the same.

Filling in all details, we find the following:

**Proposition 1.** *Consider list prices $P_L$ and $P_H$, with $0 < P_L < P_H \leq 1$. In the equilibrium of stage 2, firm $i \in \{L, H\}$ sets its retail price equal to $P_i$ with probability $\sigma_i$ and otherwise draws it from some common distribution $F(p)$ on $[\underline{p}, P_L]$, where $\sigma_L$, $\sigma_H$, $F(p)$, $\underline{p}$, and equilibrium profits $\pi_L$ and $\pi_H$ are given by:*

| *Case* *for* | $A$ $R \leq R_0$ | $B$ $R \in (R_0, R_1)$ | $C$ $R \geq R_1$ |
|---|---|---|---|
| $\sigma_L$ | $\frac{\alpha_L - \alpha_H}{1-\alpha_H}$ | $\frac{\alpha_H(R-1)}{1-\alpha_H-\alpha_L}$ | $1$ |
| $\sigma_H$ | $0$ | $\frac{(1-\alpha_H)\alpha_H R-(1-\alpha_L)\alpha_L}{(1-\alpha_L)(1-\alpha_H-\alpha_L)}$ | $1$ |
| $F(p)$ | $\frac{1-\alpha_H-\alpha_L P_L/p}{1-\alpha_H-\alpha_L}$ | $\frac{1-\alpha_L-\alpha_H P_H/p}{1-\alpha_L-\alpha_H R}$ | |
| $\underline{p}$ | $\frac{\alpha_L}{1-\alpha_H} P_L$ | $\frac{\alpha_H}{1-\alpha_L} P_H$ | |
| $\pi_L$ | $\alpha_L P_L$ | $\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L} P_H$ | $(1-\alpha_H)P_L$ |
| $\pi_H$ | $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L$ | $\alpha_H P_H$ | $\alpha_H P_H$ |

*with $R = P_H/P_L$; $R_0 = \frac{\alpha_L(1-\alpha_L)}{\alpha_H(1-\alpha_H)}$; $R_1 = \frac{1-\alpha_L}{\alpha_H}$; $\alpha_L = \frac{1-\lambda+\mu}{2}$; $\alpha_H = \frac{1-\lambda-\mu}{2}$.*

**Properties of the stage 2 equilibrium.** The results we derived above already allow us to pin down some interesting implications concerning the frequency and depth of discounts that firms give vis-à-vis their list price.

**Result 1.** *The minimal discount that firm $H$ offers is $P_H - P_L$.*

When firm $H$ uses a discount, it will always undercut the lower list price. As firm $L$ cannot price above its list price, firm $H$ can only possibly attract the fully informed consumers by setting a retail price lower than $p_L$. Offering any smaller discount would certainly be ineffective.

**Result 2.** *In cases A and B, firm H is more likely to offer a discount than firm L:* $\sigma_H < \sigma_L$.[13] *In case A, it always offers one.*

Intuitively, firm $L$ has more captive consumers and hence less of an incentive to try to attract the informed. This also implies that for $P_L$ sufficiently close to $P_H$, firm $L$ charges a higher price on average. Hence, the partially informed would then be better off buying from firm $H$. Figure 1 illustrates this for a specific parameter combination. Here, the expected retail price of firm $L$ exceeds that of firm $H$ whenever $R > R^*$, with $R^* \in (R_0, R_1)$.

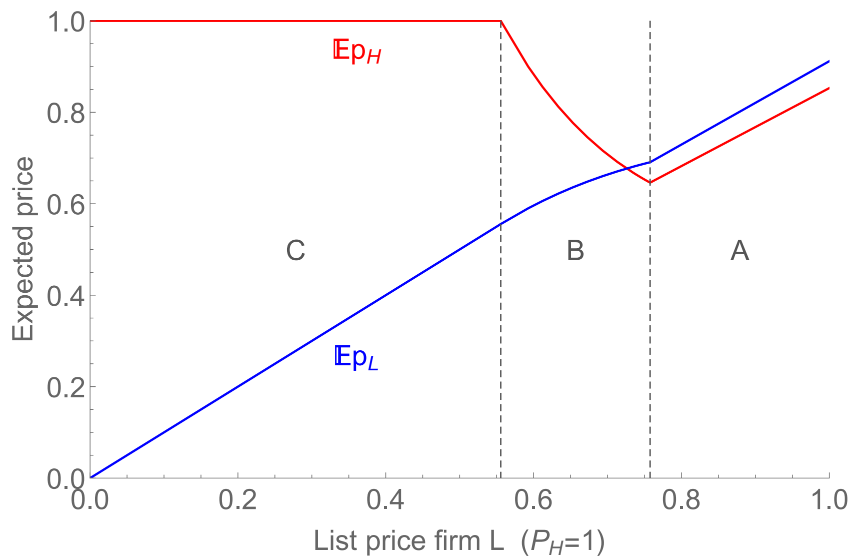
Indeed, we can show that this is always true:

**Lemma 1.** *There is a unique* $R^* \in (R_0, R_1)$ *such that the expected retail price of firm H is lower than that of firm L if and only if* $R < R^*$.

For all combinations of list prices such that $R < R^*$, the partially informed consumers thus go against their own best interest when following the simple rule of thumb of buying from the firm with the lower list price. In Section 4, we study the case when the partially informed consumers are rational and adjust their behavior accordingly. In the next two subsections, we first continue the analysis for the case of myopic partially informed consumers.

[13] In case $B$, note that $\sigma_H < \sigma_L$ reduces to $R < \frac{1-\alpha_L}{\alpha_H} = R_1$, which is true in that case.

Figure 1: Expected retail prices as a function of $P_L$, with $P_H = 1$.




Expected retail price of firm $L$ (blue line) and $H$ (red line) as a function of $P_L$ ($P_H = 1$, $\lambda = 0.2$, $\mu = 0.3$, dashed lines indicate boundaries between the cases in Proposition 1.

## 3.2 Equilibrium choice of list prices

We now solve for the equilibrium of the first stage. We refer to list prices as being *effective* if they are strictly lower than the consumers' willingness to pay. Otherwise, they have no bite.

**Equilibrium properties.** It is easy to see that in any candidate pure-strategy equilibrium at least one firm would be better off slightly undercutting the list price of its rival.[14] Using fairly standard arguments, we can then show the following:

[14]From Proposition 1, firm $L$'s equilibrium profit is weakly increasing in $P_L$ and strictly so if $R \leq R_0$. Hence, an equilibrium with $P_L^* < P_H^*$ fails to exist, as firm $L$ is better off setting $P_L$ closer to $P_H^*$. If both firms set $P^* > 0$, each has profits $\pi^* = \frac{1-\lambda}{2} P^*$. A firm that undercuts $P^*$ ends up as firm $L$ in Case A of Proposition 1, which yields deviation profits arbitrarily close to $\alpha_L P^* = \frac{1-\lambda+\mu}{2} P^* > \pi^*$, so this deviation is profitable. But $P^* = 0$ cannot be an equilibrium either: deviating to a higher price then yields positive profits.

**Proposition 2.** *Suppose the partially informed consumers are myopic. Any symmetric equilibrium then has firms sampling list prices from an atomless CDF $G(P)$ with support $[\underline{P}, 1]$, where $\underline{P} \in \left[\frac{\alpha_H}{1-\alpha_H}, \frac{1}{R_0}\right)$.*

Hence, as in Varian (1980), firms mix across list prices on some interval $[\underline{P}, 1]$, where the upper bound is given by consumers' willingness to pay. The lower bound is always such that Case B as defined in Proposition 1 can occur.[15] List prices below $\frac{\alpha_H}{1-\alpha_H}$ are dominated by setting $P_i = p_i = 1$.

We can now show the following:

**Proposition 3.** *If consumers are myopic, then in equilibrium, effective list prices are always used. The possibility to use list prices strictly decreases average equilibrium prices and profits. An upper bound on profits is given by*

$$\bar{\pi} \equiv \min\left\{\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H}, \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}\right\}. \tag{4}$$

That list prices are used in equilibrium follows directly from the observation that firms use mixed strategies. That they decrease profits can be understood as follows. Firms compete for partially informed consumers with list prices; the lower the list price, the more likely a firm is to attract those consumers. However, list prices put a ceiling on retail prices, so their use pushes down firms' feasible pricing ranges, resulting in lower transaction prices on average. Firms would like to commit not to use list prices, yet have a unilateral incentive to do so. Thus, this is a prisoner's dilemma.

Partially informed consumers can indeed have a stark impact on firms' equilibrium profits. Since $\alpha_H = \frac{1-\lambda-\mu}{2}$, the profit bound in (4) tends to zero as $\mu \to 1-\lambda$ so that the number of uninformed consumers goes to zero. Hence, as in Varian (1980), having uninformed consumers is necessary for firms to make positive profits.

[15] Since $\overline{P}/\underline{P} > R_0$, there is always a positive probability that $P_H/P_L > R_0$.

**Equilibrium characterization.** Note that Proposition 3 does not pin down equilibrium profits. In case A of Proposition 1, the profits of a firm setting $P_i = 1$ depend on the list price of its rival. Hence, equilibrium profits cannot be directly determined. The different intervals in Proposition 1 yield a second complication in deriving the mixed-strategy equilibrium. Profits of a firm depend not only on whether its list price is higher or lower than its rival's, but also on which case in Proposition 1 occurs. This greatly complicates matters. To get some more grip on the equilibrium distribution of list prices, we proceed as follows. Using Proposition 1, the expected profits of a firm charging $P$ equal

$$\Pi(P) = \overbrace{G\left(\frac{P}{R_0}\right)\alpha_H P}^{P \text{ highest, case B or C}} + \overbrace{\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}\int_{\frac{P}{R_0}}^{P} s dG(s)}^{P \text{ highest, case A}} + \overbrace{[G(PR_0) - G(P)]\alpha_L P}^{P \text{ lowest, case A}}$$
$$+ \underbrace{\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}\int_{PR_0}^{PR_1} s dG(s)}_{P \text{ lowest, case B}} + \underbrace{[1 - G(PR_1)](1-\alpha_H)P}_{P \text{ lowest, case C}}. \quad (5)$$

This can be seen as follows. If firm $i$ sets some list price $P_i$, firm $j$ may set a lower $P_j$ such that $P_i/P_j > R_0$. Given that $P_j$ is drawn from $G$, the probability that this happens is $G(P_i/R_0)$. If it does, we are in case B or C in Proposition 1, and firm $i$ has profits $\alpha_H P_i$. This yields the first term in (5). Second, for any $P_j \in (P_i/R_0, P_i)$, we end up in case A with $i$ having the higher list price, so its profits are $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}P_j$. Integrating over all relevant $P_j$ gives the second term. The remaining terms follow in a similar fashion.

For an equilibrium, we need that the right-hand side of (5) is constant for all $P \in [\underline{P}, 1]$.[16] Taking the derivative with respect to $P$, collecting terms and simplifying, we thus require for all $P \in [\underline{P}, 1]$ that:

$$G\left(\frac{P}{R_0}\right)\alpha_H - \frac{\alpha_L(\alpha_L - \alpha_H)}{1-\alpha_H}PG'(P) + [G(PR_0) - G(P)]\alpha_L + [1 - G(PR_1)](1-\alpha_H) = 0. \quad (6)$$

We thus have to solve a *functional differential equation*, as $G'(P)$ depends not only on $G(P)$, but also on $G(P/R_0)$, $G(PR_0)$ and $G(PR_1)$. If $P > 1/R_0$, the last two terms of (6) vanish. If instead $P \in [\underline{P}, \underline{P}R_0]$, its first term vanishes. Hence, the exact differential equation we have to solve depends on the position of $P$. We thus partition the support $[\underline{P}, 1]$ into a number of intervals. In each interval, $G(P)$ is the solution to a specific differential equation. Differential equations in different intervals may depend on each other. All this makes solving the entire system very intricate.

In the simplest case, the solution consists of three parts. For that, we need in equilibrium that $\underline{P}R_1 \geq 1$ (so the last term in (6) always vanishes) and $\underline{P} > 1/R_0^2$. Equation (6) then collapses to a system of three partly interdependent first-order differential equations. Proposition 4 solves that case. It applies to a relatively large part of the parameter space, as we will show numerically.

**Proposition 4.** *Suppose the partially informed consumers are myopic. For a subset of the parameter space, the symmetric equilibrium is as follows. Firms draw their list prices from the CDF $G(P)$ with support $[\underline{P}, 1]$, where*

$$G(P) = \begin{cases} a + b_1 w\,(R_0P)^{-\frac{1-w}{k}} - b_2 w\,(R_0P)^{-\frac{1+w}{k}} & \text{if } P \in \left[\underline{P}, \frac{1}{R_0}\right) \\ 1 - \left[(1-a)(1+w) - 2b_1 w\right](R_0P)^{-\frac{1}{k}} & \text{if } P \in \left[\frac{1}{R_0}, \underline{P}R_0\right) \\ a + b_1 P^{-\frac{1-w}{k}} + b_2 P^{-\frac{1+w}{k}} & \text{if } P \in [\underline{P}R_0, 1] \end{cases} \quad (7)$$

[16] It also has to be weakly lower for all $P < \underline{P}$, but this is clearly satisfied as for all $P_i < \underline{P}$, from Proposition 1 the subgame profit $\pi_i(P_i, P_j) = \pi_L(P_i, P_j)$ is weakly increasing in $P_i$.

*with*

$$w=\sqrt{\frac{\alpha_L}{\alpha_H}};\quad a=\frac{\alpha_L}{\alpha_L-\alpha_H};\quad k=\frac{\alpha_L-\alpha_H}{1-\alpha_H};$$

$$d=(1-a)\,(1+w)\,R_0^{-\frac{1}{k}};\quad e=2wR_0^{-\frac{1}{k}};$$

$$b_1=\frac{(1-a)\left[1-(\underline{P}R_0)^{-\frac{1+w}{k}}\right]-d(\underline{P}R_0)^{-\frac{1}{k}}}{(\underline{P}R_0)^{-\frac{1-w}{k}}-(\underline{P}R_0)^{-\frac{1+w}{k}}-e(\underline{P}R_0)^{\frac{1}{k}}};\quad b_2=1-a-b_1,$$

*and* $\underline{P}$ *solves*

$$a+b_1w\left(R_0\underline{P}\right)^{-\frac{1-w}{k}}-b_2w\left(R_0\underline{P}\right)^{-\frac{1+w}{k}}=0. \tag{8}$$

*For this solution to apply, it is sufficient to have* $\lambda\geq 0.38$.

For any parameter values, we can thus try to find a numerical solution as follows. First, we use (8) to solve numerically for $\underline{P}$. If this satisfies $\underline{P}\in(1/R_0^2,1/R_0)$ and $\underline{P}R_1\geq 1$, we can determine $G(P)$ using (7). If not, an equilibrium of this particular form fails to exist. Figure 9 in Appendix A shows numerically for which values of $\lambda$ and $\mu$ this procedure yields an equilibrium. It is sufficient to have $\lambda\geq 0.38$. For parameter values not covered by Proposition 4, we use a numerical approximation to find $G(P)$ on a discretized action space. Details about this procedure can be found in Appendix B.

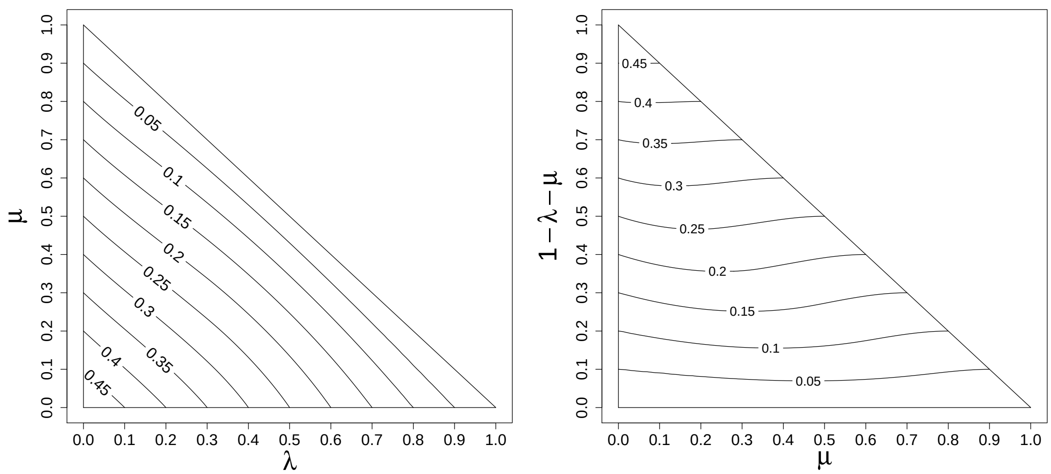
## 3.3 Welfare effects

Above we characterized the equilibrium with myopic consumers. For some parameter values, we may obtain the numerical solution implied by Proposition 4. For other parameter values, we have to do a numerical approximation. In this section, we use those results to analyze welfare effects. We focus on the comparative statics effects on profits; as all consumers buy, total welfare always equals 1 so the effects on consumer welfare are simply the opposite.

Figure 2(a) shows a contour plot of the equilibrium profits in $(\lambda,\mu)$-space. Moving up in this graph thus implies keeping the number of informed $(\lambda)$

fixed, while increasing the number of partially informed ($\mu$) at the expense of the number of uninformed ($1 - \lambda - \mu$). Similarly, moving to the right implies shifting consumers from uninformed to informed.

Figure 2: Contour plot of equilibrium profits.




(a) In $(\lambda, \mu)$-space. (b) In $(\mu, 1 - \lambda - \mu)$-space.

For values $\lambda \in \{0, 0.01, \ldots, 0.98\}$, $\mu \in \{0, 0.01, \ldots, 0.98\}$, $\lambda + \mu \leq 0.98$.

From the figure, profits are strictly decreasing in $\lambda$ and $\mu$, tending to zero as $\lambda + \mu \to 1$ (cf. the second paragraph after Proposition 3). Thus, when the share of uninformed consumers in the market decreases, firms are unambiguously worse off, no matter whether this is because the proportion of fully informed or partially informed increases.

Figure 2(b) gives the same information as Figure 2(a), but now in $(\mu, 1 - \lambda - \mu)$-space. Moving down in the graph means that uninformed consumers become fully informed. As just observed, this decreases profits. Moving to the left means that partially informed consumers become fully informed. From the graph, the effect on firm profits is non-monotonic. If the number of partially informed is low, fully informing more of them decreases profits. But if their number is high, doing so increases profits.

Note that with either $\lambda = 0$ or $\mu = 0$, we are back to Varian (1980) competition: if $\mu = 0$, competition is at the retail level; with $\lambda = 0$, it is at the list price level. If $\lambda$, $\mu > 0$, there is competition at both levels. This benefits firms relative to the case of fierce competition at either level.[17]

Summing up, we find the following:

**Numerical Result 1.** *Suppose the partially informed consumers are myopic. When uninformed consumers become either partially or fully informed, profits decrease. When partially informed consumers become fully informed, profits decrease when their share is relatively low, while profits increase when their share is relatively high.*

Armstrong (2015) gives a general analysis of models with both informed and less informed consumers ("savvy" and "non-savvy" in his more general terminology). In his analysis, there is a *search externality* when each type of consumer is better off when the number of savvy consumers increases. There is a *ripoff externality* if the opposite is true.

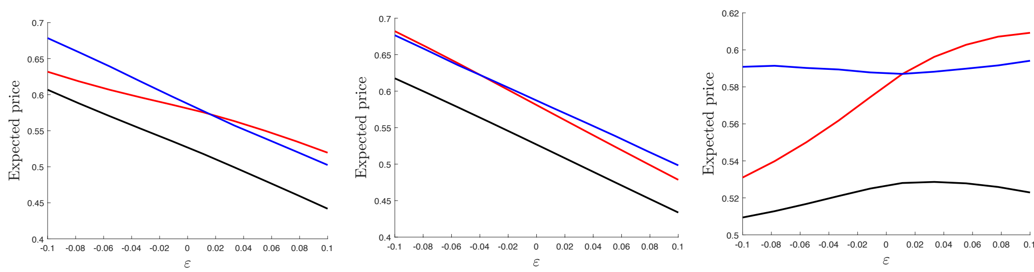
Our model not only has "savvy" and "non-savvy" consumers, but also "partly savvy" ones. It is interesting to see how an increase in savviness affects these consumer types individually. We do so for one particular parameter configuration in Figure 3.[18] The panels show the effect of fully informing uninformed consumers (a), partially informing uninformed consumers (b), and

[17]In Varian (1980), equilibrium profits are determined by the share of uninformed consumers. In our model, for a fixed share of uninformed consumers (moving on a horizontal line in Figure 2(b)), profits never fall short of those with $\mu = 0$ (on the far left of the graph) or $\lambda = 0$ (on the far right).

[18]For other parameter configurations, the graphs look qualitatively similar. Contour plots of the expected prices paid by the different consumer groups as functions of $\lambda$ and $\mu$ (similar to Figure 2) are available from the authors upon request.

fully informing partially informed consumers (c). Blue curves give the average price paid by the uninformed, red curves that paid by the partially informed, black curves that paid by the informed.

Figure 3: The effects of increasing consumer savviness.




(a) Uninformed to full. (b) Uninformed to partial. (c) Partial to full.

Average price paid by the uninformed (blue), partially uninformed (red) and informed (black) for varying $\lambda$ and $\mu$. Starting from the benchmark $\lambda = 0.25, \mu = 0.2$, the panels show the effect of (a) an increase in the fraction of fully informed by $\varepsilon$ while decreasing the fraction of uninformed by $\varepsilon$; (b) an increase in the fraction of partially informed by $\varepsilon$ while decreasing the fraction of uninformed by $\varepsilon$; (c) an increase in the fraction of fully informed by $\varepsilon$ while decreasing the fraction of partially informed by $\varepsilon$.

From the graph, informing uninformed consumers (either partially or fully) yields a search externality: due to such a change, the average price paid by all types of consumers decreases. Hence, the lower profits (and hence higher consumer surplus) we found in Figure 2 benefit all consumers. But the effect of further informing partially informed consumers is ambiguous for each consumer type. We already saw that for the aggregate effect in Figure 2(b).

From Figure 3, partially informed consumers may be worse off than uninformed consumers. The uninformed just pick a firm at random, while the partially informed choose the firm with the lower list price, which might charge a higher actual retail price on average.

# 4 Rational partially informed consumers

Above we studied the case where partially informed consumers buy from the firm with the lowest list price. Yet, from Lemma 1, this implies that they may buy from the firm with the *highest* expected retail price. Clearly, rational consumers should not behave in such a manner. In this section, we therefore study the case of rational consumers.

Suppose that indeed $\mathbb{E}p_L > \mathbb{E}p_H$ when all partially informed buy at firm $L$. Some partially informed should then switch to firm $H$. By doing so, $L$ gets fewer captive consumers, while $H$ gets more. As a result, the expected retail price of firm $L$ decreases, and that of $H$ increases. This process continues up to the point that $\mathbb{E}p_L = \mathbb{E}p_H$.[19] For a subgame equilibrium with rational consumers, we thus need:

**Definition 1.** *Given list prices $(P_L, P_H)$, an equilibrium of the retail pricing subgame with rational consumers consists of (possibly degenerate) CDFs $F_L(p_L|P_L, P_H, \theta)$ and $F_H(p_H|P_L, P_H, \theta)$, and a fraction $\theta$ of partially informed consumers that buys from firm $L$, such that*

1. *drawing $p_L$ from $F_L$ maximizes $L$'s profits given $p_H \sim F_H$ and given $\theta$;*
2. *drawing $p_H$ from $F_H$ maximizes $H$'s profits given $p_L \sim F_L$ and given $\theta$;*
3. *either one of the following conditions holds:*
   (a) *$\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ and $\theta = 1$;*
   (b) *$\mathbb{E}p_L(\theta) = \mathbb{E}p_H(\theta)$.*

[19] The only alternative would be if all partially informed buy from $H$ and $\mathbb{E}p_L > \mathbb{E}p_H$, but that cannot be part of an equilibrium either: firm $L$ would then have a lower list price *and* fewer loyal consumers, rendering its pricing more aggressive than its rival's such that $\mathbb{E}p_L < \mathbb{E}p_H$ (see also the proof of Lemma 11 in Appendix A).

We proceed as follows. Section 4.1 discusses equilibria in the retail pricing subgame. In Section 4.2, we examine the choice of list prices. Welfare implications and a comparison to the myopic case are given in Section 4.3.

## 4.1 Adjusted pricing subgames

In the myopic case, the shares of captive consumers are given by (1) and (2). As only a fraction $\theta$ of partially informed now visit firm $L$, that changes to

$$\begin{aligned}\alpha_H(\theta) &= \frac{1-\lambda-\mu}{2} + (1-\theta)\mu, \\ \alpha_L(\theta) &= \frac{1-\lambda-\mu}{2} + \theta\mu. \end{aligned} \tag{9}$$

For any $\theta$, we can directly use Proposition 1 to find the corresponding mixed-strategy equilibrium for these adapted values of $\alpha_H$ and $\alpha_L$.

To find the equilibrium with rational consumers, we thus proceed as follows:

1. If $R \geq R^* \in (R_0, R_1)$ we have $\mathbb{E}p_L(1) \leq \mathbb{E}p_H(1)$, so the equilibrium characterization in Proposition 1 still applies.

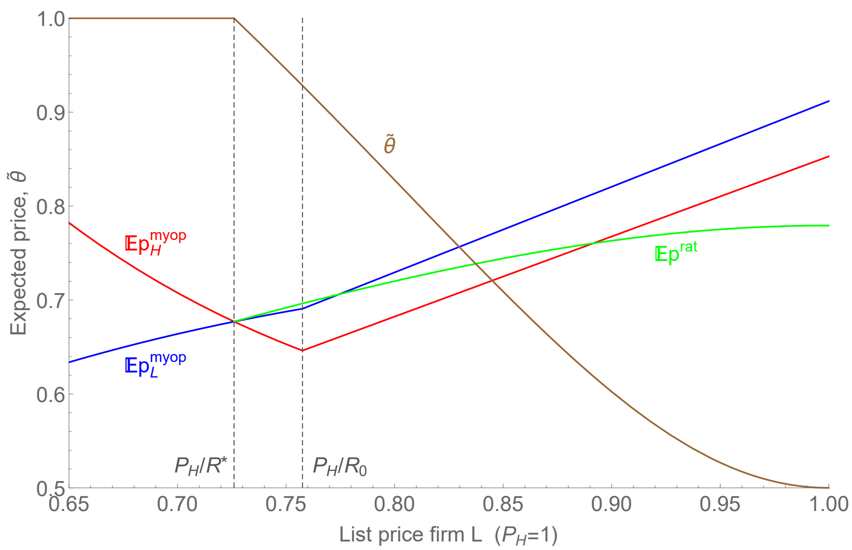
2. If $R < R^* \in (R_0, R_1)$ we have $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$. In that case, we have to find the value $\tilde{\theta}$ for which $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$.

For this procedure to work, we do need that such a $\tilde{\theta}$ always exists and is unique. This is indeed the case:

**Lemma 2.** *For any $(P_L, P_H)$ with $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$, there is a unique $\tilde{\theta} \in \left(\frac{1}{2}, 1\right)$ such that $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$.*

**Example.** To illustrate how the analysis is affected by having rational consumers, we revisit the example in Figure 1, with $\lambda = 0.2$ and $\mu = 0.3$. Figure 4

Figure 4: Expected prices with myopic and rational consumers.

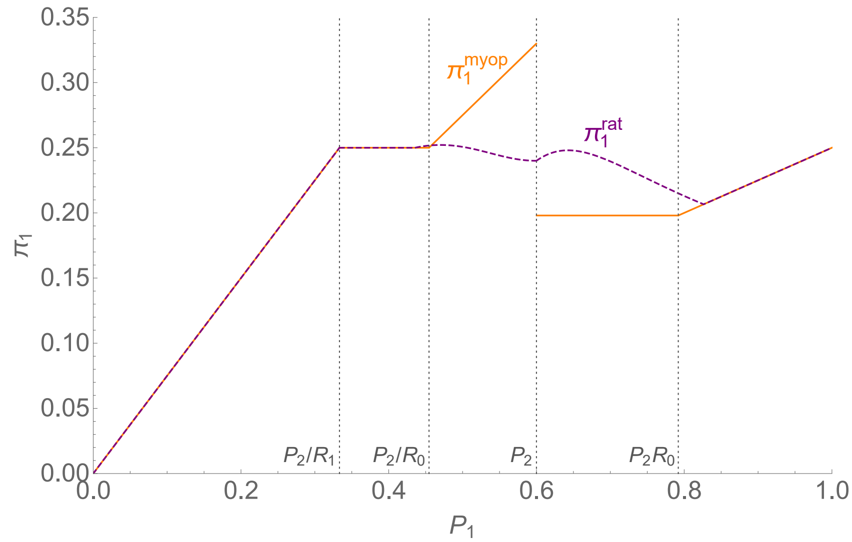



Expected price of firm $L$ and firm $H$ as a function of the list price of firm $L$ with myopic (blue and red line, respectively) and with rational partially informed consumers (green line). Also depicted: equilibrium value $\tilde{\theta}$ (brown line). The parameters used are $P_H = 1$, $\lambda = 0.2$, $\mu = 0.3$.

adds rational consumers to that picture. The blue and red lines again give the expected retail prices of $L$ and $H$ with myopic consumers, the green line that of both firms if consumers are rational. The brown line reflects the equilibrium share of partially informed $\tilde{\theta}$ that visit firm $L$. This share monotonically decreases from 1 (if $P_H/P_L = R^*$) to 1/2 (if $P_H/P_L = 1$).

Interestingly, expected prices are now higher in some pricing subgames but lower in others. With rational consumers, firms become more symmetric in their share of captive consumers. If list prices are close to each other, this leveled playing field implies more aggressive competition (cf. Narasimhan, 1988, p. 441, point 1.iii). However, if the difference in list prices is large, the playing field is already very tilted to start with. Having more captive consumers now only makes $H$ more reluctant to compete for the informed, as that requires sacrificing its relatively high margin on an increased base of captive consumers. With $H$ competing less aggressively, $L$ follows suit.

Figure 5: Profits of firm 1 with myopic and rational consumers.




Profits of firm 1 as a function of $P_1$, given $P_2 = 0.6$ ($\lambda = 0.2, \mu = 0.3$). Orange: myopic partially informed. Purple: rational partially informed.

## 4.2 Equilibrium choice of list prices

We next study how incentives in the first stage of the game are affected. To illustrate, Figure 5 shows the expected profits of firm 1 as a function of $P_1$ if $P_2 = 0.6$, again for $\lambda = 0.2$ and $\mu = 0.3$. The orange curve represents the case of myopic, the purple dashed curve that of rational consumers.

In the myopic case, slightly undercutting $P_2$ attracts *all* partially informed consumers and hence implies a discrete upward jump in profits. In the rational case, it only slightly increases the number of partially informed consumers firm 1 attracts. In the figure, the best reply of firm 1 is then to choose the local maximum between $P_2/R^*$ and $P_2$. But for slightly different parameter values, the best reply may be to choose the local maximum between $P_2$ and $P_2R^*$ – or to set $P_1 = 1$. Small changes may thus imply big shifts in the best reply of firm 1. This makes the analysis even more involved.

As can be seen in Figure 5, firm profits now are continuous, but they fail to be quasi-concave. As a result, existence of a symmetric pure-strategy equilibrium is not guaranteed. Indeed, we have the following:

**Proposition 5.** *Suppose that the partially informed consumers are rational.*

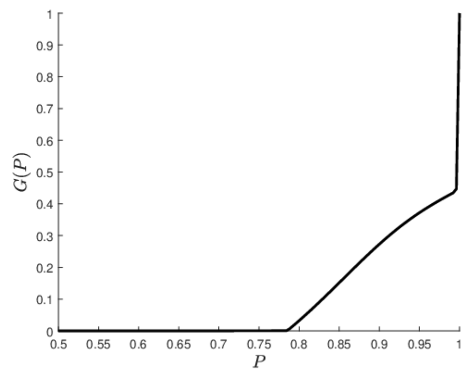
- *If* $\lambda \geq 1/3$*, there is a unique symmetric pure-strategy equilibrium in which both firms set* $P = 1$*. Hence, no effective list prices are used.*
- *If* $\lambda < 1/3$*, a symmetric pure-strategy equilibrium fails to exist, and effective list prices are used in equilibrium.*

Starting from $P_1 = P_2 = 1$, lowering one's list price has two effects. First, it increases one's share of partially informed consumers, which increases profits. But it also makes competition for the informed consumers more aggressive, which tends to decrease profits. If the number of informed consumers is sufficiently large, the second effect dominates, leaving $P = 1$ as an equilibrium. An equilibrium with $P_1 = P_2 < 1$ fails to exist: firms would then prefer to set a slightly *higher* list price than their competitor.

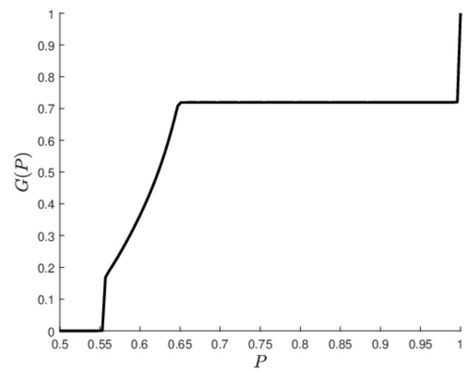
**The equilibrium for $\lambda < 1/3$.** For this case, it is hard to characterize the equilibrium choice of list prices. As always, a mixed-strategy equilibrium requires that each firm is indifferent between all list prices in its support. But, if list prices are sufficiently close to each other, we also need that shares of loyal consumers are such that expected retail prices are equalized. In turn, these endogenously determined shares affect the subgame equilibrium profits. Moreover, subgame equilibrium profits fail to be quasi-concave.

We therefore have to resort to a numerical approximation of the equilibrium mixed-strategy choice of list prices. As it turns out, the equilibrium distribution often has mass points and gaps, so we cannot use the method described in Appendix B. Instead, we use a numerical procedure based on Mangasarian and Stone (1964). Roughly, for each parameter combination, we discretize the action space, construct the respective payoff matrix, and numerically solve a quadratic programming problem.[20]

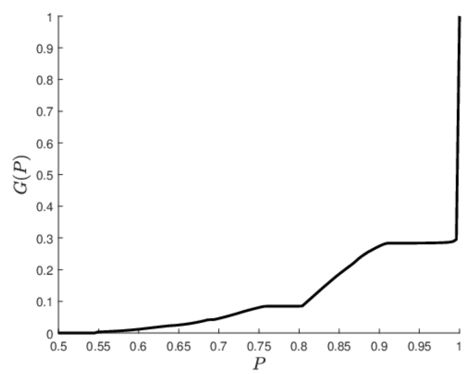
Figure 6: Approximated first-stage equilibrium CDFs.



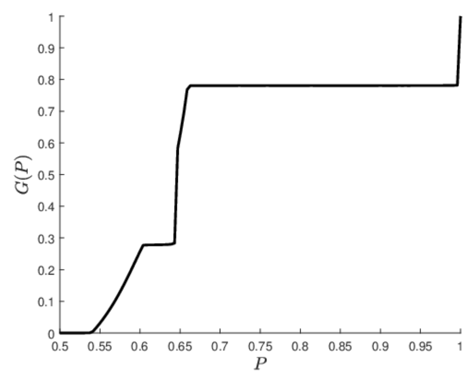

(a) $\lambda = 0.25, \mu = 0.3$




(b) $\lambda = 0.2, \mu = 0.3$




(c) $\lambda = 0.25, \mu = 0.15$




(d) $\lambda = 0.15, \mu = 0.3$

Approximated first-stage equilibrium CDFs for various parameter combinations. In each case, an equidistant grid of size 256 over the interval $[0, 1]$ was used.

[20]Further details in Heijnen (2020). The corresponding Matlab code is available upon request. We also confirmed our results using an alternative, evolutionary algorithm.

To illustrate the complexity, Figure 6 shows the equilibrium CDF $G(P)$ for four sets of parameters. The equilibrium in the top-left panel is fairly well-behaved, but does have a mass point at $P = 1$. The equilibrium in the top-right panel has two mass points: at $P = 1$ and $P \approx 0.55$. Moreover, the support has a gap at $[0.65, 1)$. In the bottom-left panel, there are two gaps but only one mass point. The bottom-right panel has two mass points and two gaps.

Even though the parameter values are close, the resulting equilibria are qualitatively different. The likely cause is that small changes in parameter values may trigger substantial differences in best replies, as we saw in the discussion of Figure 5. From our numerical results, even with $\lambda < 1/3$ firms refrain from using effective list prices (and hence set $P = 1$) with positive probability.

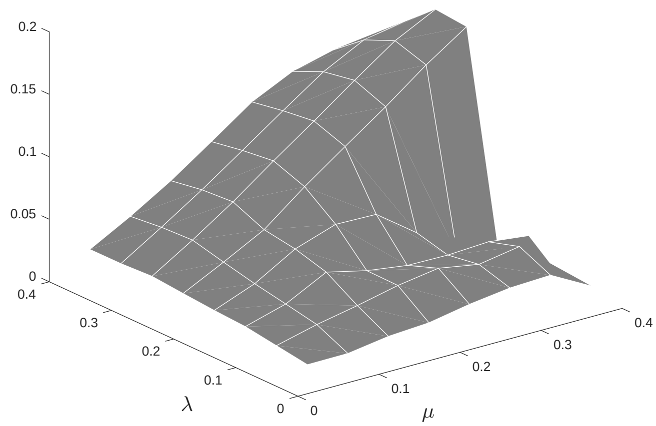
## 4.3 Welfare effects

We now consider the welfare implications of the possibility to use list prices when consumers are rational. From Proposition 5, if $\lambda \geq 1/3$, firms set $P = 1$ so the model coincides with the Varian (1980) benchmark. For $\lambda < 1/3$, we find a result equivalent to Proposition 3:

**Proposition 6.** *If the partially informed consumers are rational, then the possibility to use list prices has no effect on equilibrium profits if* $\lambda \geq 1/3$*, but strictly decreases average equilibrium prices and profits if* $\lambda < 1/3$*.*

With myopic consumers, we had from Proposition 3 that the possibility to use list prices always yields lower profits. For $\lambda \geq 1/3$, we thus immediately have that profits are higher and consumer surplus is lower when consumers are rational rather than myopic. For $\lambda < 1/3$, an analytical comparison is not feasible and we have to resort to our numerical analysis. Figure 7 gives the differences in expected profits between the case of rational and that of myopic consumers. It turns out that that difference is always positive. We thus have:

Figure 7: Profit differences between the rational case and the myopic case.




For both cases, profits are approximated by solving for the Nash equilibrium of a discretized version of the game. The discretization uses an equidistant grid of size 256 over the interval $[0, 1]$. Positive values mean that profits are higher in the case of rational consumers.

**Numerical Result 2.** *With rational consumers, profits are strictly higher and consumer surplus strictly lower than with myopic consumers.*

Why the comparison is so hard to do analytically can also be understood from Figure 4: for some combinations of list prices, expected prices are higher in the rational case, while for others, they are lower. The net effect then depends on how often certain combinations are chosen in equilibrium.

Armstrong (2015) makes a distinction between consumers that are "savvy" since they are well-informed, and those that are *strategically savvy* in the sense that they have a good understanding of the game being played. Hence, our partially informed consumers are strategically naive if myopic, and strategically savvy if rational. Our analysis then implies a ripoff externality in this dimension: when the partially informed become strategically savvy, the consumers end up paying a higher price on average.[21] Hence, consumers as a whole would be better off if they could commit as a group to use the simple rule of thumb.

As noted, the type of equilibrium we end up in (and hence the equilibrium profit) is highly sensitive to parameter values. This also implies that we cannot conduct accurate comparative statics, as we did in Figure 2. Neither is it possible in this context to study the effect of an increase in consumer savviness.

# 5 List prices and the scope for collusion

In a number of cases, antitrust authorities have been concerned about collusion in list prices, and how that could affect transaction prices. A notable example is the truck cartel in the EU, where six producers of trucks agreed (amongst others) upon harmonizing list prices between 1997 and 2011,[22] and were fined a total of 3.7 billion euro – still the highest EU cartel fine to date. For a detailed discussion of many relevant cartel cases in Europe and the US, see Boshoff and Paha (2021). Cartelists have argued that list price collusion is really harmless, as higher list prices will simply be offset by higher rebates, leaving transaction prices unaffected. Antitrust authorities often argue otherwise, but tend to be vague concerning the theory of harm.[23] Our model may provide exactly that.

[21] In the main text we consider cases where partially informed consumers either are all myopic, or all rational. It is straightforward to allow for a fraction $\kappa \in (0,1)$ that are rational. Note that with $\kappa = 1$, an equilibrium fraction $\tilde{\theta}$ visits firm $L$, while with $\kappa = 0$ we impose $\theta = 1$. If $1-\kappa \leq \tilde{\theta}$ we have the same solution as with $\kappa = 1$: having $\kappa \geq 1-\tilde{\theta}$ rational partially informed consumers is enough to reach the fully rational outcome. If $\kappa < 1 - \tilde{\theta}$, we get the solution described in Proposition 1, but with $\tilde{\alpha}_L = \frac{1-\lambda-\mu}{2} + (1-\kappa)\mu$ and $\tilde{\alpha}_H = \frac{1-\lambda-\mu}{2} + \kappa\mu$.

[22] See `https://ec.europa.eu/competition/antitrust/cases/dec_docs/39824/39824_8750_4.pdf` and `https://ec.europa.eu/competition/antitrust/cases/dec_docs/39824/39824_8754_5.pdf`.

[23] Again, see Boshoff and Paha (2021).

Most theoretical contributions on list-price collusion consider list prices as a starting point that serves as the basis for price negotiations between the producer and its customers (see e.g. Harrington and Ye (2019) or Gill and Thanassoulis (2016)). Our mechanism is completely different. In our model, consumers differ in the amount of information they have, and list prices are used to try to attract partially informed consumers, while still retaining downward pricing flexibility to compete for fully informed consumers.

In this section, we thus study collusion with list prices. First, we discuss whether in our model successful collusion in list prices indeed leads to higher transaction prices, as often argued by antitrust authorities. Second, we study whether the possibility to use list prices in itself facilitates collusion in a world where collusion is feasible in both the list price as well as the retail price stage. Third, we study how firms' ability to collude, either fully or only on list prices, is affected by consumer information.

First of all, from Propositions 3 and 6, we immediately have:

**Proposition 7.** *Collusion in list prices leads to higher retail prices on average.*

With perfect collusion on list prices, firms set them equal to 1. From Propositions 3 and 6, this increases average prices compared to the case that effective list prices are used.[24]

Next, we consider whether the ability to use list prices facilitates optimal collusion. In the remainder of this section, we consider an infinitely repeated version of the baseline model of Section 2, where firms are infinitely lived but a new cohort of consumers arrives each period. Firms use a common discount factor $\delta \in (0, 1)$. We only consider the more tractable case of myopic consumers and restrict attention to collusion via grim trigger strategies. For such collusion

[24] Recall that with rational partially informed consumers, this holds when $\lambda < 1/3$.

to be sustainable, we need $\delta \geq \overline{\delta} \equiv \frac{\pi^D - \pi^C}{\pi^D - \pi^N}$, with $\pi^N$ denoting Nash profits, $\pi^C$ collusive profits, and $\pi^D$ optimal defection profits in the stage game.

Without the possibility of list prices, we are back to Varian (1980), and $\pi^N = \frac{1-\lambda}{2}$. The cartel price is $p_i = 1$, so $\pi^C = 1/2$ and $\pi^D = \frac{1+\lambda}{2}$. Hence, our benchmark critical discount factor is $\overline{\delta}_{bench} = 1/2$.

If firms can use list prices and consumers are myopic, we can use Appendix B to numerically find the stage-game Nash profit $\pi^N$ of the full game. Perfect collusion requires $P_i = p_i = 1$. This again implies $\pi^C = 1/2$.

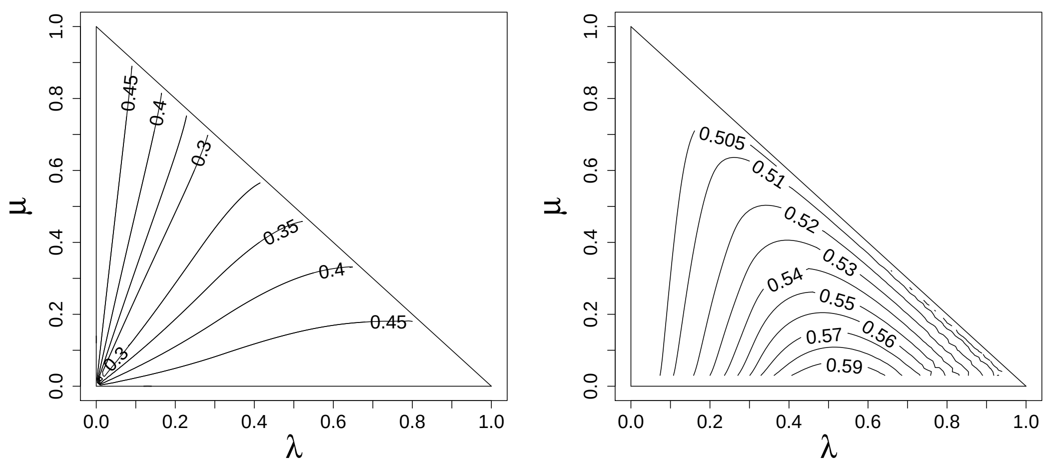
For defection profits, note that firms can either defect in the list-price stage or in the retail-price stage. When they do in the list-price stage, we assume that reversion to the Nash equilibrium already takes place in the retail-pricing stage of the same period. From Proposition 1, the best defection then is to marginally undercut $P_i = 1$, which yields $\pi^D = \frac{1-\lambda+\mu}{2}$. In the retail-pricing stage, the best defection is to marginally undercut $p_i = 1$, which yields $\pi^D = \frac{1+\lambda}{2}$. Firms thus prefer to defect in the list-price stage if $\mu > 2\lambda$.

Figure 8(a) shows a contour plot of the resulting critical discount factor $\overline{\delta}$ in $(\lambda, \mu)$-space. For all combinations of $\lambda$ and $\mu$, it lies below that of the benchmark case (i.e, 1/2), implying that the ability to use list prices facilitates collusion. We can indeed prove this formally:

**Proposition 8.** *When the partially informed consumers are myopic, the possibility to use list prices facilitates collusion.*

This can be seen as follows. First, the ability to use list prices does not affect collusive profits. Second, from Proposition 3, it lowers Nash profits, making the loss when defecting from a collusive agreement more severe. Defection profits are either unaffected by the use of list prices (if the share of informed consumers is relatively high), or they increase only moderately as a defection

Figure 8: Contour plot of critical discount factors to support collusion.




(a) Full collusion.

(b) List-price collusion.

For values $\lambda \in \{0, 0.01, \dots, 0.98\}$, $\mu \in \{0, 0.01, \dots, 0.98\}$, $\lambda + \mu \leq 0.98$.

in the list price stage immediately provokes aggressive (Nash) pricing in the subsequent pricing stage. The more severe punishment dominates, implying that collusion is facilitated.

We now turn to the last question: how is the scope for collusion affected by consumer information? First, if firms can collude in both stages of the game, it is apparent from the contour plot in Figure 8(a) that the effect of increasing consumer information[25]) is ambiguous. Next consider a scenario where firms can *only* collude on list prices. Figure 8(b) shows a contour plot of the critical discount factor $\overline{\delta}$ in $(\lambda, \mu)$-space in this scenario. We then find:

**Numerical Result 3.** *When consumers are myopic and firms can only collude on list prices, collusion is facilitated when the share of partially informed consumers increases at the expense of uninformed consumers.*

[25]i.e. partially or fully informing uninformed consumers, or fully informing partially informed consumers – which corresponds to moving to the south-east in the figure.

Defection profits increase in the share of partially informed consumers, but the punishment also becomes more severe, as list price competition then becomes more intense. The latter effect dominates, hence the result.

# 6 Conclusion

In this paper, we have studied a simple homogeneous-goods duopoly in the spirit of Varian (1980) where firms first set list prices and then set possibly discounted retail prices. Next to the informed and uninformed consumers in Varian (1980), we also introduced *partially informed consumers* whose purchase decision is solely influenced by list prices. The main insights from our analysis are as follows.

First, for given list prices, whenever price discounts are granted, the firm with the higher list price gives deeper and more frequent discounts. This is because a successful discount must at least beat the other firm's list price. Moreover the firm with the higher list price has a smaller share of captive consumers, which makes attracting informed customers relatively more lucrative.

Second, if the partially informed consumers simply buy from the firm with the lower list price, a pure-strategy equilibrium in list prices fails to exist. This is because by slightly undercutting the competitor's list price, a firm could then capture the entire mass of partially informed consumers. However, for list prices that are relatively close, the firm with the lower list price would then also have a higher retail price on average. Rational partially informed consumers understand this, and hence do not simply go to the firm with the lower list price. But this implies that list price competition for rational consumers is less fierce, exactly because slightly undercutting the competitor no longer captures all partially informed consumers.

Third, firms would be better off *not* setting list prices, as their use leads to lower transaction prices on average. This is a prisoners' dilemma as each firm has an incentive to attract the partially informed by setting a lower list price. This is particularly true in the case of myopic consumers, and less so with rational consumers, for the reason set out above. It also implies that successful collusion on list prices leads to higher retail prices. Using list prices facilitates collusion, as competition is fiercer when a cartel breaks down.

Fourth, in the myopic case, having more informed consumers tends to lower average prices. But this is only true if uninformed consumers become better informed, making the market more competitive. If partially informed consumers become fully informed, the result may be different. Expected prices are lowest if there is fierce competition at either the list price stage (so the number of partially informed is high) or at the retail price stage (so their number is low). For intermediate values, competition is not too fierce at either level, and expected retail prices are higher as a result.

A considerable limitation of our model is its lack of tractability. In general, a closed-form solution for the list-price equilibrium cannot be obtained. In the myopic case, we can pin down the equilibrium explicitly (up to the lower bound of its support) for parts of the parameter space, and our equilibrium characterization results at least permit the use of a simple and robust numerical procedure to compute the mixed-strategy equilibrium. In the rational case, list prices will often not be used, but if they are, few characterization results are available. Our numerical results indicate highly irregular and parameter-sensitive equilibrium behavior in this case. This reduces the accuracy of our numerical results, limiting the level of detail of the analyses we can conduct.

There are several potential directions for future research. First, it would be interesting to endogenize consumer group sizes by explicitly modelling advertising decisions that inform consumers, either partially or fully. Second, search costs may be explicitly introduced into the model, hence endogenizing the behavior of different types of consumers. Third, studying the consequences of some degree of product differentiation may be informative, though this is likely to complicate the analysis even further.

# References


Simon P. Anderson, Alicia Baik, and Nathan Larson. Price Discrimination in the Information Age: Prices, Poaching, and Privacy with Personalized Targeted Discounts. CEPR Discussion Papers 13793, C.E.P.R. Discussion Papers, June 2019. URL `https://ideas.repec.org/p/cpr/ceprdp/13793.html`.

Enrique Andreu, Damien Neven, and Salvatore Piccolo. Delegated Sales, Agency Costs and the Competitive Effects of List Price. CSEF Working Papers 573, Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy, July 2020. URL `https://ideas.repec.org/p/sef/csefwp/573.html`.

Mark Armstrong. Search and Ripoff Externalities. *Review of Industrial Organization*, 47(3):273–302, 2015.

Jeffrey Banks and Sridhar Moorthy. A model of price promotions with consumer search. *International Journal of Industrial Organization*, 17:371–398, 1999.

Helmut Bester and Emmanuel Petrakis. Coupons and oligopolistic price discrimination. *International Journal of Industrial Organization*, 14(2):227–242, 1996.

Willem Boshoff, Stefan Frübing, and Kai Hüschelrath. Information exchange through non-binding advance price announcements: an antitrust analysis. *European Journal of Law and Economics*, 45(3):439–468, 2018.

Willem H. Boshoff and Johannes Paha. List price collusion. *Journal of Industry, Competition and Trade*, 21(3):393–409, 2021.

Lisa Bruttel. The Effects of Recommended Retail Prices on Consumer and Retailer Behaviour. *Economica*, 85(339):649–668, 2018.

Stefan Buehler and Dennis L. Gärtner. Making Sense of Nonbinding Retail-Price Recommendations. *American Economic Review*, 103(1):335–359, 2013.

Yongmin Chen and Robert W. Rosenthal. Asking Prices as Commitment Devices. *International Economic Review*, 37(1):129–155, 1996a.

Yongmin Chen and Robert W. Rosenthal. On the use of ceiling-price commitments by monopolists. *The RAND Journal of Economics*, 27(2):207–220, 1996b.

Michael Choi, Anovia Yifan Dai, and Kyungmin Kim. Consumer search and price competition. *Econometrica*, 86(4):1257–1281, 2018.

Partha Dasgupta and Eric Maskin. The Existence of Equilibrium in Discontinuous Economic Games, I: Theory. *The Review of Economic Studies*, 53 (1):1–26, 1986.

Antón García Díaz, Roberto Hernán González, and Praveen Kujal. List pricing and discounting in a Bertrand-Edgeworth duopoly. *International Journal of Industrial Organization*, 27(6):719–727, 2009.

Yucheng Ding and Tianle Zhang. Price-directed consumer search. *International Journal of Industrial Organization*, 58:106–135, 2018.

David Gill and John Thanassoulis. Competition in Posted Prices with Stochastic Discounts. *The Economic Journal*, 126(594):1528–1570, 2016.

Marco A. Haan, J.L. Moraga González, and Vaiva Petrikaitė. A model of directed consumer search. *International Journal of Industrial Organization*, 61:223–255, 2018.

Joseph E. Harrington and Lixin Ye. Collusion through Coordination of Announcements. *The Journal of Industrial Economics*, 67(2):209–241, 2019.

Paul Heidhues and Botond Kőszegi. Regular prices and sales. *Theoretical Economics*, 9:217–251, 2014.

Pim Heijnen. On the Computation of Equilibrium in Discontinuous Economic Games. Som research report 2020008 eef, University of Groningen, 2020.

Maarten C.W. Janssen, José Luis Moraga-González, and Matthijs R. Wildenbeest. Truly costly sequential search and oligopolistic pricing. *International Journal of Industrial Organization*, 23:451–466, 2005.

Dmitry Lubensky. A model of recommended retail prices. *The RAND Journal of Economics*, 48(2):358–386, 2017.

O.L. Mangasarian and H. Stone. Two-Person Nonzero-Sum Games and Quadratic Programming. *Journal of Mathematical Analysis and Applications*, 9(3):348–355, 1964.

David P. Myatt and David Ronayne. A Theory of Stable Price Dispersion. mimeo, 2019.

Chakravarthi Narasimhan. Competitive Promotional Strategies. *The Journal of Business*, 61(4):427–449, 1988.

Martin Obradovits. Austrian-style gasoline price regulation: How it may backfire. *International Journal of Industrial Organization*, 32:33–45, 2014.

Johannes Paha. Anchoring, Reference Prices, and List Price Collusion. VfS Annual Conference 2019 (Leipzig): 30 Years after the Fall of the Berlin Wall - Democracy and Market Economy 203625, Verein für Socialpolitik / German Economic Association, January 2019.

Clemens Puppe and Stephanie Rosenkranz. Why Suggest Non-Binding Retail Prices? *Economica*, 78(310):317–329, 2011.

Ram C. Rao. Pricing and Promotions in Asymmetric Duopolies. *Marketing Science*, 10(2):131–144, 1991.

Andrew Rhodes and Chris M. Wilson. False advertising. *The RAND Journal of Economics*, 49(2):348–369, 2018.

Greg Shaffer and Z. John Zhang. Competitive Coupon Targeting. *Marketing Science*, 14(4):395–416, 1995.

Hal Varian. A Model of Sales. *American Economic Review*, 70(4):651–659, 1980.

# Appendix A: Technical Proofs

*Proof of Proposition 1.* Case C has been established in the main text, with profits following trivially. We prove Cases A and B using the following lemmas.

**Lemma 3.** *If $R \leq R_0$, the pricing subgame has the following unique mixed-strategy equilibrium. Firm $H$ draws its price from the CDF*

$$F_H(p) = 1 - \frac{\alpha_L\left(\frac{P_L}{p} - 1\right)}{1 - \alpha_L - \alpha_H}$$

*with support $\left[\frac{\alpha_L}{1-\alpha_H}P_L, P_L\right)$. Firm $L$ sets $p_L = P_L$ with probability*

$$\sigma_L = \frac{\alpha_L - \alpha_H}{1 - \alpha_H},$$

*and draws its price from $F_H(p)$ with probability $1 - \sigma_L$. Expected profits are*

$$\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}P_L \qquad \textit{and} \qquad \pi_L = \alpha_L P_L.$$

*Proof of Lemma 3.* We only prove that this is an equilibrium. Uniqueness can be established with the usual arguments, available upon request.

Suppose firm $L$ plays according to the Lemma. If firm $H$ sets $p_H \in \left[\frac{\alpha_L}{\alpha_L+\lambda}P_L, P_L\right)$, it has attracts the informed with probability $\sigma_L + (1-\sigma_L)(1 - F_L(p_H))$ yielding profits

$$p_H\left[\alpha_H + \lambda(\sigma_L + (1-\sigma_L)(1 - F_L(p_H)))\right] = \frac{(\alpha_H + \lambda)\alpha_L}{\alpha_L + \lambda}P_L.$$

Setting $p_H < \frac{\alpha_L}{\alpha_L+\lambda}P_L$ makes no sense, as it already attracts all informed. Charging $p_H = P_L$ makes no sense either: this is a mass point for $L$ so undercutting it increases profits. As any $p_H > P_L$ will not attract the informed, the best such deviation is $p_H = P_H$. This yields $\alpha_H P_H$, which does not exceed $\pi_H$ as $R \leq R_0$. Hence, $H$ has no profitable deviation.

Suppose firm $H$ plays according to the Lemma. If $L$ sets $p_L \in \left[\frac{\alpha_L}{\alpha_L+\lambda}P_L, P_L\right]$, it attracts the informed with probability $1 - F_H(p_L)$ yielding profits

$$\pi_L(p_L; P_L, P_H) = p_L\left[\alpha_L + \lambda(1 - F_H(p_L))\right] = \alpha_L P_L.$$

Setting $p_L < \frac{\alpha_L}{\alpha_L+\lambda}P_L$ makes no sense, as this already attracts all informed for sure. Firm $L$ cannot price above $P_L$. Hence it has no profitable deviation.

Lastly, all equilibrium objects are well-behaved, since clearly $\sigma_L \in (0,1)$, while $F_i(\frac{\alpha_L}{\alpha_L+\lambda}P_L) = 0$, $F_i(P_L) = 1$, and $\frac{dF_i(p)}{dp} = \frac{\alpha_L P_L}{p^2\lambda} > 0$. ■

**Lemma 4.** *If $R \in (R_0, R_1)$, the pricing subgame has the following unique mixed-strategy equilibrium. Firm $H$ sets $p_H = P_H$ with probability*

$$\sigma_H = \frac{(1-\alpha_H)\alpha_H R}{(1-\alpha_H-\alpha_L)(1-\alpha_L)} - \frac{\alpha_L}{1-\alpha_H-\alpha_L},$$

*and with probability $1-\sigma_H$ draws its price from the CDF*

$$F_H(p) = \frac{1-\alpha_L-\alpha_H\left(\frac{P_H}{p}\right)}{1-\alpha_L-\alpha_H R}$$

*with support $\left[\frac{\alpha_H}{1-\alpha_L}P_H, P_L\right)$. Firm $L$ sets $p_L = P_L$ with probability*

$$\sigma_L = \frac{\alpha_H(R-1)}{1-\alpha_H-\alpha_L},$$

*and draws its price from $F_H(p)$ with probability $1-\sigma_L$. Expected profits are*

$$\pi_H = \alpha_H P_H \qquad \text{and} \qquad \pi_L = \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P_H.$$

*Proof of Lemma 4.* Again, we only prove that this is an equilibrium. Details concerning uniqueness are available upon request. Suppose $L$ plays according to the lemma. If $H$ sets $p_H \in \left[\frac{\alpha_H}{\alpha_H+\lambda}P_L, P_L\right)$, it has expected profits

$$p_H\left[\alpha_H + \lambda(\sigma_L + (1-\sigma_L)(1-F_L(p_H)))\right] = \alpha_H P_H.$$

If it sets $p_H = P_H$, its profits are also $\alpha_H P_H$. Setting $p_H < \frac{\alpha_H}{\alpha_H+\lambda} P_L$ makes no sense, as it already attracts all informed for sure. Setting $p_H = P_L$ makes no sense either as this is a mass point for $L$ so undercutting it increases profits. As any $p_H > P_L$ will not attract the informed, any price in $(P_L, P_H)$ yields lower profits than $p_H = P_H$. Hence, firm $H$ has no profitable deviation.

Suppose firm $H$ plays according to the lemma. If $L$ sets $p_L \in \left[\frac{\alpha_H}{\alpha_H+\lambda} P_L, P_L\right]$, it has expected profits

$$p_L \left[\alpha_L + \lambda(\sigma_H + (1-\sigma_H)(1 - F_H(p_L)))\right] = \frac{(\alpha_L + \lambda)\alpha_H}{\alpha_H + \lambda} P_H.$$

Setting $p_L < \frac{\alpha_H}{\alpha_H+\lambda} P_L$ makes no sense as this already attracts all informed for sure. It cannot price above $P_L$. Hence, firm $L$ has no profitable deviation.

It remains to verify that all equilibrium objects are well-behaved. First, it is easy to check that $\sigma_H \in (0,1)$ if $R \in (R_0, R_1)$. Second, $\sigma_L > 0$ as $R > R_0 > 1$, while $\sigma_L < 1$ follows from $R < R_1$. Lastly, $F_i(\frac{\alpha_H}{\alpha_H+\lambda} P_L) = 0$, $F_i(P_L) = 1$, and

$$\frac{dF_i(p)}{dp} = \frac{\alpha_H P_H}{p^2[\lambda - \alpha_H(R-1)]} > 0,$$

where the inequality follows from $R < R_1$. ■

This completes the proof of Proposition 1. ■

*Proof of Lemma 1.* For high $R$ we are in case $C$ where $p_L^* < p_H^*$. For low $R$ we are in case $A$ where $\sigma_H = 0$ and $\sigma_L > 0$ imply $\mathbb{E}p_L > \mathbb{E}p_H$. For case $B$ we will show that $\mathbb{E}p_L$ strictly increases in $P_L$, and $\mathbb{E}p_H$ strictly decreases in $P_L$. Continuity then implies that there must be a unique $P_L \in (P_H/R_1, P_H/R_0)$ where $\mathbb{E}p_L = \mathbb{E}p_H$ which established the result. More precisely, in case B,

$$\begin{aligned}\mathbb{E}p_L &= \sigma_L P_L + (1-\sigma_L)\int_{\underline{p}}^{P_L} p dF(p) \\ &= \frac{\alpha_H}{1-\alpha_L-\alpha_H}\left[P_H - P_L + P_H \log\left(\frac{1-\alpha_L}{\alpha_H}\frac{P_L}{P_H}\right)\right], \qquad (10)\end{aligned}$$

while

$$\mathbb{E}p_H = \sigma_H P_H + (1-\sigma_H)\int_{\underline{p}}^{P_L} p dF(p)$$
$$= \frac{P_H\left[\alpha_H(1-\alpha_H)P_H/P_L - \alpha_L(1-\alpha_L) + (1-\alpha_H)\alpha_H \log\left(\frac{1-\alpha_L}{\alpha_H}\frac{P_L}{P_H}\right)\right]}{(1-\alpha_L)(1-\alpha_L-\alpha_H)}. \tag{11}$$

Hence

$$\frac{d\mathbb{E}p_L}{dP_L} = \frac{\alpha_H}{1-\alpha_L-\alpha_H}(P_H/P_L - 1) > 0,$$

and

$$\frac{d\mathbb{E}p_H}{dP_L} = -\frac{P_H}{P_L}\left[\frac{\alpha_H(1-\alpha_H)}{(1-\alpha_L)(1-\alpha_L-\alpha_H)}\right](P_H/P_L - 1) < 0.$$

■

*Proof of Proposition 2.* Note that existence follows from Dasgupta and Maskin (1986). We first establish a number of lemmas. First, any firm can always choose to set $p_i = P_i = 1$ and sell to at least its captive consumers. Hence

**Lemma 5.** *Each firm has expected profit of at least* $\alpha_H$ *in equilibrium.*

The firm with the lower $P_i$ sells at most $1-\alpha_H$ at price of at most $P_i$. If $P_i < \alpha_H/(1-\alpha_H)$, profits are below the $\alpha_H$ it obtains by $p_i = P_i = 1$. Hence

**Lemma 6.** *In equilibrium, no firm sets* $P$ *below* $\underline{P}_{min} \equiv \frac{\alpha_H}{1-\alpha_H} > 0$.

**Lemma 7.** $G(\cdot)$ *is atomless.*

*Proof.* If $G(\cdot)$ has an an atom at $P^*$, firms set $P^*$ with some probability $\beta > 0$, yielding profits $\frac{1-\lambda}{2}P^*$. Both then prefer setting $P^* - \epsilon$ with probability $\beta$. ■

**Lemma 8.** *In equilibrium* $\overline{P}/\underline{P} > R_0$, *so case* $B$ *can always occur.*

*Proof.* Suppose to the contrary $\overline{P}/\underline{P} \leq R_0$. Then $\pi_L = \alpha_L P_L$ and $\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_L < \pi_L$. With $G$ atomless, setting $\underline{P}$ yields $\alpha_L \underline{P}$. But then, if $\underline{P} > 0$, any $P_i > \underline{P}$ in the equilibrium support yields a lower profit of $\pi_H = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} \underline{P}$ which cannot be true in equilibrium. Also, $\underline{P} \neq 0$ due to Lemma 6. ■

**Lemma 9.** $\bar{P} = 1$.

*Proof.* Suppose $\bar{P} < 1$. If firm $i$ now deviates to some $P_i \in (\overline{P}, 1]$, it makes either $\alpha_H P_i$ (if the other firm sets $P_j \leq P_i/R_0$ and we are in case $B$ or $C$) or $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_i$ (if $P_j \in (P_i/R_0, \overline{P}]$ and we are in case $A$). Hence, we can write

$$\pi_i(P_i) = G\left(\frac{P_i}{R_0}\right)\alpha_H P_i + \int_{\frac{P_i}{R_0}}^{\overline{P}} \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P dG(P).$$

Taking the derivative with respect to $P_i$ yields

$$\begin{aligned}\pi_i'(P_i) &= G\left(\frac{P_i}{R_0}\right)\alpha_H + G'\left(\frac{P_i}{R_0}\right)\alpha_H \frac{P_i}{R_0} - \frac{1}{R_0}\left[\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}\right] G'\left(\frac{P_i}{R_0}\right)\frac{P_i}{R_0} \\ &= G\left(\frac{P_i}{R_0}\right)\alpha_H.\end{aligned}$$

Hence, $\lim_{\epsilon \downarrow 0} \pi_i'(\overline{P} + \epsilon) = G(\overline{P}/R_0)\alpha_H > 0$, where the inequality follows from Lemma 8. But then, setting $P$ marginally above $\overline{P} < 1$ would be a profitable defection, so this cannot be part of an equilibrium. ■

**Lemma 10.** *There are no gaps in $G(\cdot)$.*

*Proof.* Suppose $G$ does contain gaps, and the highest is $(a, b)$ for some $a < b < 1$, with $G(a) = G(b) < 1$. From Proposition 1, if $P_j < P_i$, we have that $\pi_i$ is either $\alpha_H P_i$ (if $P_i/P_j \geq R_0$), or $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} P_j$ (if $P_i/P_j < R_0$). Hence, conditional on $P_j < P_i$, $\pi_i$ is weakly increasing in $P_i$.

If instead $P_i < P_j$, $\pi_i$ is either $\alpha_L P_i$ (if $P_j/P_i < R_0$), or $\frac{(1-\alpha_H)\alpha_H}{1-\alpha_L} P_j$ (if $R_0 \leq P_j/P_i < R_1$), or $(1-\alpha_H)P_i$ (if $P_j/P_i \geq R_1$). Again, conditional on

$P_j > P_i$, $\pi_i$ is weakly increasing in $P_i$. But then we would have $\pi_i(a) < \pi_i(b)$: for $P_i \in (\max\{a, b/R_0\}, b)$, increasing $P_i$ increases $\pi_i$ when $P_j \in [b, P_i R_0)$, which happens with positive probability since by assumption $(a, b)$ is the highest gap in $G$. This cannot be the case in equilibrium. ■

Taken together, the lemmas above prove Proposition 2. ■

*Proof of Proposition 3.* We first derive the upper bound on profits, then show that these are strictly lower than profits in the Varian (1980) case.

From Proposition 2, both $P = 1$ and $P = 1/R_0$ are in the equilibrium support, and $G$ is atomless. Suppose first firm $i$ sets $P_i = 1$. With $P_j \leq 1$ and $\alpha_H < \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}$, Proposition 1 implies that $\pi_i$ cannot exceed $\frac{(1-\alpha_L)\alpha_L}{1-\alpha_H}$. Now suppose firm $i$ sets $P_i = 1/R_0$. If $P_j > P_i$ we are in case A and $\pi_i = \alpha_L P_i$. If $P_j < P_i$, there are two possibilities. In case A of Proposition 1, $\pi_i = \frac{1-\alpha_L}{1-\alpha_H}\alpha_L P_j < \alpha_L P_i$. In case B and C, $\pi_i = \alpha_H P_i < \alpha_L P_i$. Hence, an upper bound on profits is given by $\alpha_L P_i = \frac{\alpha_L}{R_0} = \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}$.

Recall next that with Varian (1980) competition, equilibrium profits are $\pi^* = \frac{1-\lambda}{2}$. For the statement on profits, it thus suffices to show that

$$\min\left\{\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H}, \frac{\alpha_H(1-\alpha_H)}{1-\alpha_L}\right\} < \frac{1-\lambda}{2}. \tag{12}$$

Using $\alpha_L = \frac{1-\lambda+\mu}{2}$ and $\alpha_H = \frac{1-\lambda-\mu}{2}$, some straightforward algebra implies $\frac{\alpha_L(1-\alpha_L)}{1-\alpha_H} < \frac{1-\lambda}{2}$ if and only if $\lambda < \frac{1+\mu}{3}$, while $\frac{\alpha_H(1-\alpha_H)}{1-\alpha_L} < \frac{1-\lambda}{2}$ if and only if $\lambda > \frac{1-\mu}{3}$. Since at least one of these conditions is always satisfied, (12) holds.

■

*Proof of Proposition 4.* Suppose that in equilibrium $\underline{P}R_1 \geq 1$ and $\underline{P} > 1/R_0^2$. Partition the support into three non-empty intervals $\mathcal{I}_1 = [\underline{P}, 1/R_0)$, $\mathcal{I}_2 = [1/R_0, \underline{P}R_0)$ and $\mathcal{I}_3 = [\underline{P}R_0, 1]$. Denote the distribution function in interval $i \in$

$\{1, 2, 3\}$ by $G_i$ and the corresponding density function by $g_i$. Using Proposition 2, we must have that $G_1(\underline{P}) = 0$, $G_1(1/R_0) = G_2(1/R_0)$, $G_2(\underline{P}R_0) = G_3(\underline{P}R_0)$, and $G_3(1) = 1$. Note that for prices $P \in \mathcal{I}_1$, we have $PR_0 \in \mathcal{I}_3$ and $P/R_0 < \underline{P}$. Hence, using $\underline{P}R_1 \geq 1$, (6) then reduces to

$$G_1(P) + \left(\frac{\alpha_L - \alpha_H}{1 - \alpha_H}\right) P g_1(P) - G_3(PR_0) = 0. \tag{13}$$

For prices $P \in \mathcal{I}_2$, we have $PR_0 > 1$ and $P/R_0 < \underline{P}$. Thus (6) then reduces to

$$1 - G_2(P) - \left(\frac{\alpha_L - \alpha_H}{1 - \alpha_H}\right) P g_2(P) = 0. \tag{14}$$

Finally, for prices $P \in \mathcal{I}_3$, we have $PR_0 > 1$ and $P/R_0 \in \mathcal{I}_1$, so (6) reduces to

$$1 - G_3(P) - \left(\frac{\alpha_L - \alpha_H}{1 - \alpha_H}\right) P g_3(P) + G_1(P/R_0)\frac{\alpha_H}{\alpha_L} = 0. \tag{15}$$

We can use (14) to solve for $G_2$. Next, (15) allows us to write $G_1$ in terms of $G_3$ and $g_3$, and $g_1$ in terms of $g_3$ and $g_3'$. Plugging these into (13) yields a differential equation for $G_3$ that can be solved analytically. We can then use (13) to write $G_3$ in terms of $G_1$ and $g_1$, and $g_3$ in terms of $g_1$ and $g_1'$. Plugging these into (15) yields a differential equation for $G_1$ that can also be solved analytically. From (14) $G_2$ has the form

$$G_2(P) = 1 - B_0 P^{-\frac{1}{k}}, \tag{16}$$

where

$$k = \frac{\alpha_L - \alpha_H}{1 - \alpha_H} \in (0, 1), \tag{17}$$

and $B_0$ is a coefficient to be determined.

To solve for $G_1$ and $G_3$, we first introduce the variable $z \equiv P/R_0$, which we substitute in (15) to obtain

$$G_1(z) = \frac{\alpha_L}{\alpha_H}\left[kR_0 z g_3(R_0 z) - (1 - G_3(R_0 z))\right]. \tag{18}$$

Taking the derivative with respect to $z$ and simplifying yields

$$g_1(z) = \frac{\alpha_L}{\alpha_H}\left[R_0 g_3(R_0 z)(k+1) + kR_0^2 z g_3'(R_0 z)\right].$$

Plugging these expressions for $G_1$ and $g_1$ into (13) yields, after simplification,

$$1 - G_3(P)\left(\frac{\alpha_L - \alpha_H}{\alpha_L}\right) - k(2+k)Pg_3(P) - k^2P^2g_3'(P) = 0. \tag{19}$$

We conjecture that $G_3(P)$ has the following functional form:

$$G_3(P) = a + b_1P^{c_1} + b_2P^{c_2},$$

such that

$$Pg_3(P) = b_1c_1P^{c_1} + b_2c_2P^{c_2}$$

and

$$P^2g_3'(P) = b_1c_1(c_1-1)P^{c_1} + b_2c_2(c_2-1)P^{c_2}.$$

Substituting these expressions and comparing coefficients, we find

$$a = \frac{\alpha_L}{\alpha_L - \alpha_H}, \tag{20}$$

$$c_{1,2} = -\frac{1}{k}(1 \pm w),$$

with $w \equiv \sqrt{\frac{\alpha_H}{\alpha_L}}$, while $b_1$ and $b_2$ are still unspecified.

Note that $c_1$ and $c_2$ are given by $k^2c_i^2 + 2kc_i + \frac{\alpha_L - \alpha_H}{\alpha_L} = 0$, $i = 1, 2$. Necessarily $c_1 \neq c_2$: otherwise $G_3(P) = a + bP^c$ cannot yield a general solution to a second-order ordinary differential equation. For concreteness, let

$$c_1 = -\frac{1-w}{k}, \tag{21}$$

$$c_2 = -\frac{1+w}{k}. \tag{22}$$

Hence we have

$$G_3(P) = a + b_1P^{-\frac{1-w}{k}} + b_2P^{-\frac{1+w}{k}}. \tag{23}$$

Using $G_3(1) = 1$, we require

$$b_2 = b_2(b_1) = 1 - a - b_1. \tag{24}$$

We next introduce the variable $q \equiv PR_0$, which we substitute in (13) to obtain

$$G_3(q) = G_1\left(\frac{q}{R_0}\right) + \frac{kq}{R_0} g_1\left(\frac{q}{R_0}\right). \tag{25}$$

After taking the derivative with respect to $q$, we obtain

$$g_3(q) = \frac{1}{R_0} g_1\left(\frac{q}{R_0}\right) + \frac{k}{R_0} g_1\left(\frac{q}{R_0}\right) + \frac{kq}{R_0^2} g_1'\left(\frac{q}{R_0}\right).$$

Plugging these expressions for $G_3$ and $g_3$ into (15) and simplifying yields

$$1 - G_1(P)\left(\frac{\alpha_L - \alpha_H}{\alpha_L}\right) - k(2+k)Pg_1(P) - k^2P^2g_1'(P) = 0. \tag{26}$$

This differential equation coincides with that for $G_3(P)$ above. Hence, we need

$$G_1(P) = a + \beta_1 P^{c_1} + \beta_2 P^{c_2},$$

with $a$, $c_1$ and $c_2$ specified above. From (23) and (18) we have

$$G_1(P) = a + \left[b_1 R_0^{c_1} w\right] P^{c_1} + \left[-b_2(b_1) R_0^{c_2} w\right] P^{c_2},$$

which pins down $\beta_1$ and $\beta_2$ as functions of $b_1$:

$$\begin{aligned} \beta_1(b_1) &= b_1 R_0^{c_1} w, \\ \beta_2(b_1) &= -b_2(b_1) R_0^{c_2} w. \end{aligned} \tag{27}$$

The requirement that $G_1(1/R_0) = G_2(1/R_0)$ pins down $B_0$ as a function of $b_1$:

$$B_0(b_1) = \frac{1 - a - \beta_1(b_1)R_0^{-c_1} - \beta_2(b_1)R_0^{-c_2}}{R_0^{\frac{1}{k}}}.$$

Inserting $\beta_1(b_1)$, $\beta_2(b_1)$ and using $b_1 + b_2(b_1) = 1 - a$ then yields

$$B_0(b_1) = \frac{(1-a)(1+w) - 2b_1 w}{R_0^{\frac{1}{k}}}. \tag{28}$$

Now $G_2(\underline{P}R_0) = G_3(\underline{P}R_0)$ yields an expression for $b_1$, conditional on $\underline{P}$:

$$1 - B_0(b_1) \cdot (\underline{P}R_0)^{-\frac{1}{k}} = a + b_1(\underline{P}R_0)^{c_1} + (1 - a - b_1)(\underline{P}R_0)^{c_2}.$$

As this is linear in $b_1$, we can directly solve for $b_1$, given $\underline{P}$:

$$b_1(\underline{P}) = \frac{(1-a)[1 - (\underline{P}R_0)^{c_2}] - d(\underline{P}R_0)^{-\frac{1}{k}}}{(\underline{P}R_0)^{c_1} - (\underline{P}R_0)^{c_2} - e(\underline{P}R_0)^{-\frac{1}{k}}}, \tag{29}$$

where

$$d = (1-a)(1+w)\, R_0^{-\frac{1}{k}}; e = 2wR_0^{-\frac{1}{k}}. \tag{30}$$

The final step to solve for equilibrium is the consistency requirement that

$$G_1(\underline{P}; a, \beta_1(b_1(\underline{P})), \beta_2(b_1(\underline{P}))) = 0. \tag{31}$$

Taken together, this implies the result.

Figure 9 shows for which parameters the above procedure indeed yields a solution for the first-stage price distribution. In particular, the procedure works for all $\lambda$ sufficiently large ($\lambda \gtrapprox 0.38$), irrespective of $\mu$. ■

*Proof of Lemma 2.* We first establish the following:

**Lemma 11.** *If $\tilde{\theta}$ exists, it is such that $\tilde{\alpha}_H$ is a root of*

$$h(\tilde{\alpha}_H; R) = \left(1 - \frac{1-\tilde{\alpha}_H}{\lambda + \tilde{\alpha}_H}\right) \log\left(\frac{\lambda + \tilde{\alpha}_H}{\tilde{\alpha}_H R}\right) + \frac{1-\lambda}{\tilde{\alpha}_H} - \frac{1}{R} - \frac{1-\tilde{\alpha}_H}{\lambda + \tilde{\alpha}_H} R.$$

*Moreover, we have (1) $\tilde{\theta} > 1/2$, (2) $\frac{\partial h}{\partial \tilde{\alpha}_H} < 0$, (3) $\frac{\partial h}{\partial R} < 0$, and (4) $\frac{\partial^2 h}{\partial \tilde{\alpha}_H \partial R} > 0$.*

*Proof.* As noted, we need $\mathbb{E}p_L(\tilde{\theta}) = \mathbb{E}p_H(\tilde{\theta})$. Equating (10) and (11) in the proof of Lemma 1 and simplifying, we thus need $\tilde{\theta}$ to be such that

$$1 - \frac{1}{R} - \frac{\tilde{\alpha}_L - \tilde{\alpha}_H}{1 - \tilde{\alpha}_L} \log\left(\frac{1-\tilde{\alpha}_L}{\tilde{\alpha}_H}\frac{1}{R}\right) - \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L} R + \frac{\tilde{\alpha}_L}{\tilde{\alpha}_H} = 0.$$

Using $\tilde{\alpha}_L = 1 - \lambda - \tilde{\alpha}_H$ yields the expression for $h(\tilde{\alpha}_H; R)$.

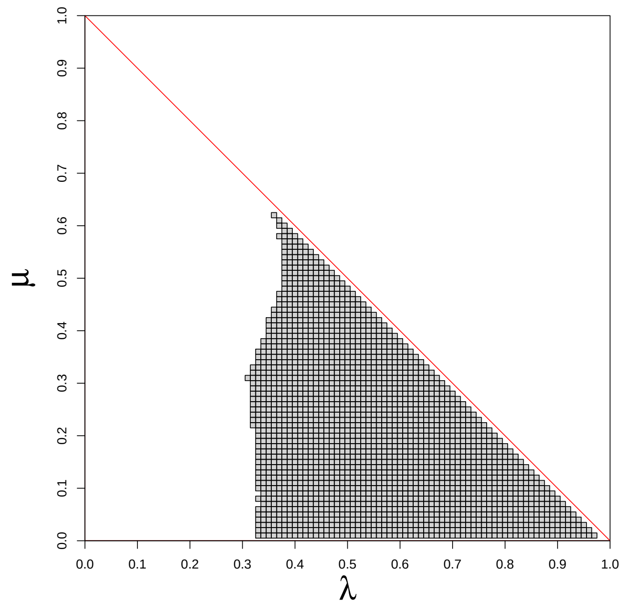



Figure 9: Parameter combinations ($\lambda \in \{0.01, 0.02, ..., 0.97\}$, $\mu \in \{0.01, 0.02, ..., 0.97\}$, $\lambda + \mu \leq 0.98$). Each square corresponds to a feasible parameter combination, centered at the respective parameters. Black squares indicate parameter combination for which Proposition 4 yields a valid solution.

To prove (1), we show that $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ for $\theta \leq \frac{1}{2}$. In case C of Proposition 1, this is true as both firm charge the list price. In case B,

$$\mathbb{E}p_L(\theta) = (1 - \sigma_L) \int_{\underline{p}}^{P_L} p dF(p) + \sigma_L P_L$$
$$\mathbb{E}p_H(\theta) = (1 - \sigma_H) \int_{\underline{p}}^{P_L} p dF(p) + \sigma_H P_H.$$

With $P_H > P_L$, if $\sigma_H \geq \sigma_L$, we have $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$. Now $\sigma_H \geq \sigma_L$ requires

$$(1 - \tilde{\alpha}_H)\tilde{\alpha}_H R - (1 - \tilde{\alpha}_L)\tilde{\alpha}_L \geq (1 - \tilde{\alpha}_L)\tilde{\alpha}_H R - (1 - \tilde{\alpha}_L)\tilde{\alpha}_H,$$

which implies

$$(\tilde{\alpha}_L - \tilde{\alpha}_H)\tilde{\alpha}_H R \geq (\tilde{\alpha}_L - \tilde{\alpha}_H)(1 - \tilde{\alpha}_L).$$

With $\theta \le 1/2$, we have $\tilde{\alpha}_L - \tilde{\alpha}_H \le 0$, so this implies

$$R \le \frac{1-\tilde{\alpha}_L}{\tilde{\alpha}_H} = \tilde{R}_1,$$

which is true since we are in Case B.

To prove the other claims, note that

$$\frac{\partial h}{\partial R} = -\frac{(R-1)\left(R\left(1-\alpha_H\right)+\lambda+\alpha_H\right)}{R^2\left(\lambda+\alpha_H\right)} < 0$$

and hence

$$\frac{\partial^2 h}{\partial \tilde{\alpha}_H \partial R} = \frac{(1+\lambda)(R-1)}{R\left(\lambda+\tilde{\alpha}_H\right)^2} > 0,$$

which establishes claims (3) and (4). Note next that

$$\frac{\partial h}{\partial \tilde{\alpha}_H} = \frac{\lambda-\lambda^2-2\lambda\tilde{\alpha}_H}{\tilde{\alpha}_H\left(\lambda+\tilde{\alpha}_H\right)^2} + \left(\ln\frac{\lambda+\tilde{\alpha}_H}{\tilde{\alpha}_H R}\right)\frac{1+\lambda}{\left(\lambda+\tilde{\alpha}_H\right)^2} - \frac{1}{\tilde{\alpha}_H^2}(1-\lambda) + R\frac{1+\lambda}{\left(\lambda+\tilde{\alpha}_H\right)^2}.$$

Claim (4) then implies that if $\partial h/\partial\tilde{\alpha}_H$ is negative at $R_1$, then it is negative for all $R \in (R_0, R_1)$. We thus need[26]

$$\left.\frac{\partial h}{\partial \tilde{\alpha}_H}\right|_{R=R_1} = \frac{\lambda-\lambda^2-2\tilde{\alpha}_H\lambda}{(\lambda+\tilde{\alpha}_H)^2} - \frac{1-\lambda}{\tilde{\alpha}_H} + \frac{1+\lambda}{\lambda+\tilde{\alpha}_H} < 0.$$

Multiplying by $\tilde{\alpha}_H(\lambda+\tilde{\alpha}_H)^2$, we require

$$\tilde{\alpha}_H(\lambda-\lambda^2-2\tilde{\alpha}_H\lambda) - (1-\lambda)(\lambda+\tilde{\alpha}_H)^2 + \tilde{\alpha}_H(1+\lambda)(\lambda+\tilde{\alpha}_H) < 0,$$

which simplifies to

$$(2\tilde{\alpha}_H - 1) + \lambda < 0.$$

Using $\lambda = 1-\tilde{\alpha}_L-\tilde{\alpha}_H$, this simplifies to $\tilde{\alpha}_H < \tilde{\alpha}_L$ which is true for $\tilde{\theta} > \frac{1}{2}$. ■

To establish Lemma 2, note that from the proof of Lemma 11, we have $\mathbb{E}p_L(\theta) < \mathbb{E}p_H(\theta)$ if $\theta < \frac{1}{2}$. By construction $\mathbb{E}p_L(1) > \mathbb{E}p_H(1)$. Since $\mathbb{E}p_L(\theta)$ and $\mathbb{E}p_H(\theta)$ are continuous in $\theta$, this establishes existence.

[26]Note that the term containing the logarithm drops at $R = R_1$.

For uniqueness $\mathbb{E}p_L - \mathbb{E}p_H$ needs to be monotonic in $\theta$ for $\theta \in (\frac{1}{2}, 1)$. Now

$$\frac{dh}{d\tilde{\theta}} = \frac{dh}{d\tilde{\alpha}_H}\frac{d\tilde{\alpha}_H}{d\tilde{\theta}} = -\mu \frac{dh}{d\tilde{\alpha}_H},$$

where we use $d\tilde{\alpha}_H/d\tilde{\theta} = -\mu$. Hence it is sufficient to have that $h$ is monotonic in $\tilde{\alpha}_H$, which is true from Claim 2 in Lemma 11. ■

*Proof of Proposition 5.* We check whether both firms can set the same $P$ in equilibrium. This would imply per-firm profits of $\frac{1-\lambda}{2}P$. We proceed as follows:

1. Suppose $i$ deviates to a lower $P_i$ with $P/P_i \geq R^*$. From Lemma 1, we then have $\mathbb{E}p_L < \mathbb{E}p_H$, so all partially informed visit $i$. This yields profits

$$\Pi_i^d(P_i; P) = \begin{cases} (1-\alpha_H)P_i & \text{if} \quad P_i \leq P/R_1 \\ \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P & \text{if} \quad P_i \in (P/R_1, P/R^*]. \end{cases} \tag{32}$$

   With $(1-\alpha_H)P_i$ strictly increasing in $P_i$, it is never a best reply to set $P_i < P/R_1$. Hence, the best possible defection in this range yields $\Pi_i^d = \frac{(1-\alpha_H)\alpha_H}{1-\alpha_L}P$. This is weakly lower than $\frac{1-\lambda}{2}P$ whenever $\lambda \geq \frac{1-\mu}{3}$. Hence, for $\lambda \geq \frac{1-\mu}{3}$, firm $i$ weakly prefers $P_i = P$ over any $P_i \leq P/R^*$.

2. Suppose $i$ deviates to a lower $P_i$ with $P/P_i < R^*$. From Lemma 1, not all partially informed consumers go to $i$. Moreover, $i$ and $j$ must have the same expected retail price. From (9), this yields

$$\Pi_i^d(P_i; P) = \frac{(1-\tilde{\alpha}_H)\tilde{\alpha}_H}{1-\tilde{\alpha}_L}P = \frac{(1-\tilde{\alpha}_H)\tilde{\alpha}_H}{\tilde{\alpha}_H + \lambda}P.$$

   This implies that

$$\frac{\partial \Pi_i^d(P_i; P)}{\partial P_i} = \frac{\partial \Pi_i^d(P_i; P)}{\partial \tilde{\alpha}_H}\frac{d\tilde{\alpha}_H(P_i)}{dP_i} = -\left[1 - \frac{(1+\lambda)\lambda}{(\lambda + \tilde{\alpha}_H)^2}\right] P \frac{d\tilde{\alpha}_H}{dP_i}. \tag{33}$$

   Using the implicit function theorem,

$$\frac{d\tilde{\alpha}_H}{dR} = -\frac{\partial h/\partial R}{\partial h/\partial \tilde{\alpha}_H} < 0,$$

as follows from claims (2) and (3) of Lemma 11. With $R = P/P_i$,

$$\frac{d\tilde{\alpha}_H}{dP_i} = \frac{d\tilde{\alpha}_H}{dR}\frac{dR}{dP_i} > 0.$$

Hence, $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} \geq 0$ for all $P_i \in (P/R^*, P)$ if the bracketed term in (33) is weakly negative in this interval. This term strictly increases in $\tilde{\alpha}_H$, which strictly increases in $P_i$. We thus need $\lim_{P_i \to P}\left[1 - \frac{(1+\lambda)\lambda}{(\lambda+\tilde{\alpha}_H)^2}\right] \leq 0$. As $\lim_{P_i \to P} \tilde{\alpha}_H = \frac{1-\lambda}{2}$, this is equivalent to

$$1 - \frac{4\,(1+\lambda)\,\lambda}{(1+\lambda)^2} \leq 0,$$

which reduces to $\lambda \geq 1/3$. Hence, if $\lambda \geq 1/3$, we have $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} \geq 0$ for $P_i \in (P/R^*, P)$, so $i$ sets $P_i = P$. If $\lambda < 1/3$, we have $\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} < 0$ for all $P_i$ sufficiently close below $P$, so firm $i$ undercuts $P$.

3. Suppose $P < 1$ and firm $i$ deviates to a higher $P_i$ with $P_i/P < R^*$. That yields $\Pi_i^d(P_i;P) = \tilde{\alpha}_H P_i$ , so

$$\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i} = \frac{d\tilde{\alpha}_H}{dP_i}P_i + \tilde{\alpha}_H = \frac{d\tilde{\alpha}_H}{dR}\frac{dR}{dP_i}P_i + \tilde{\alpha}_H = -\frac{\partial h/\partial R}{\partial h/\partial \tilde{\alpha}_H}\frac{P_i}{P} + \tilde{\alpha}_H.$$

Evaluated at $P_i = P$, the first term is zero, hence

$$\left.\frac{\partial \Pi_i^d(P_i;P)}{\partial P_i}\right|_{P_i=P} = \frac{1-\lambda}{2} > 0.$$

4. For $\lambda \geq 1/3$, steps 1 and 2 imply that $P = 1$ is an equilibrium, while step 3 implies that an equilibrium cannot have $P < 1$.

5. For $\lambda < 1/3$, step 2 implies that any firm wants to deviate from any symmetric equilibrium.

■

*Proof of Proposition 6.* The case $\lambda \geq 1/3$ follows from Proposition 5. For $\lambda < 1/3$, first note that with rational consumers we can never end up in case A of Proposition 1, as Lemma 1 implies that in case A we always have $\mathbb{E}p_L > \mathbb{E}p_H$. Hence only cases B and C are relevant.

Suppose that $P_L$ and $P_H$ are such that $R < R^*$. From Proposition 1,

$$\begin{aligned} \pi_L + \pi_H &= \tilde{\alpha}_H P_H \left(1 + \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L}\right) \\ &\leq \tilde{\alpha}_H \left(1 + \frac{1-\tilde{\alpha}_H}{1-\tilde{\alpha}_L}\right) = \frac{(1+\lambda)[1-\lambda-\mu(2\tilde{\theta}-1)]}{1+\lambda-\mu(2\tilde{\theta}-1)}, \end{aligned} \tag{34}$$

where $\tilde{\theta}$ again equalizes expected retail prices. The right-hand side decreases in $\tilde{\theta}$. Hence, an upper bound can be found by setting $\tilde{\theta} = 1/2$. This implies

$$\pi_L + \pi_H \leq 1 - \lambda.$$

Now suppose $P_L$ and $P_H$ are such that $R \in [R^*, R_1)$. From Proposition 1, (34) then still applies, but with $\tilde{\theta} = 1$ (and hence $\alpha_H$ and $\alpha_L$ rather than $\tilde{\alpha}_H$ and $\tilde{\alpha}_L$, respectively). Hence, we now have, $\pi_L + \pi_H < 1 - \lambda$.

Finally, suppose $P_L$ and $P_H$ are such that $R \geq R_1$. From Proposition 1,

$$\begin{aligned} \pi_L + \pi_H &= (1-\alpha_H)P_L + \alpha_H P_H \\ &\leq (1-\alpha_H)\frac{P_H}{R_1} + \alpha_H P_H = \alpha_H P_H \left(1 + \frac{1-\alpha_H}{1-\alpha_L}\right) \\ &\leq \alpha_H \left(1 + \frac{1-\alpha_H}{1-\alpha_L}\right) < 1 - \lambda, \end{aligned}$$

where the last inequality again follows from the same argument used for (34). Without list prices, we are in the Varian case and total profits equal $1 - \lambda$. This establishes the result. ■

*Proof of Proposition 8.* With $\mu \leq 2\lambda$, the optimal defection is in the retail-pricing stage. The result then follows immediately from Proposition 3. For

$\mu > 2\lambda$, the optimal defection is in the list-price stage, with $\pi^D = \frac{1-\lambda+\mu}{2} = \alpha_L$. We thus need to establish that $\overline{\delta} = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} < 1/2$, which is equivalent to

$$\pi^N < 2\pi^C - \pi^D = 1 - \alpha_L.$$

This is indeed true: from Proposition 3 we know that $\pi^N < \frac{\alpha_L(1-\alpha_L)}{1-\alpha_H} < 1 - \alpha_L$, where the last inequality follows from $\frac{\alpha_L}{1-\alpha_H} < 1$. ■

# Appendix B: Numerical analysis

Our numerical approach proceeds as follows. For any $(\lambda, \mu)$, discretize the action space by breaking down the candidate support $[\underline{P}_{min}, 1]$ into $l$ actions $a_1, ..., a_l$, where $a_k$ ($k \in \{1, ..., l\}$) implies choosing $P = \underline{P}_{min} + (k-1)\left(\frac{1-\underline{P}_{min}}{l-1}\right)$. Then use Proposition 1 to construct a $l \times l$ payoff matrix $A$, with $a_{ij}$ $i$'s expected profit when choosing $a_i$ while $j$ chooses $a_j$. We set $a_{ii} = \frac{(1-\alpha_L)\alpha_L}{1-\alpha_H} a_i$ on the main diagonal. Hence the row player is treated as having a strictly higher list price in case of a tie. This slightly increases incentives to compete, but improves accuracy by creating just a single discontinuity in payoffs around $a_i = a_j$.

Let $f_k$ denote the $(l-k+1) \times 1$ vector describing the frequency distribution of actions $(a_k, ..., a_l)$. Let $\iota_k$ denote a vector of ones of corresponding length. Finally, let $A_k$ be the $(l-k+1) \times (l-k+1)$ submatrix of $A$ with rows $k$ to $l$ and columns $k$ to $l$. Then, for given $k$, the following linear system in $f_k$ is a candidate equilibrium with expected profit $\gamma$:

$$A_k f_k = \gamma \cdot \iota_k \quad (35)$$

$$\iota_k' f_k = 1. \quad (36)$$

Here $a_k$ serves as a guess for the lower bound $\underline{P}$ of $G(P)$. Equation (35) then states that for given support $\{a_k, ..., a_l\}$, each action yields the same payoff $\gamma$ (as $G(P)$ cannot contain gaps), while (36) requires frequencies to sum to one.

To numerically approximate the equilibrium, we use the following algorithm. First, take $k = 1$. Second, solve the above linear system of $l - k + 2$ equations in $l - k + 2$ unknowns for $f_k$ and $\gamma$. If $A_k$ is invertible and $\iota_k' A_k^{-1} \iota_k \neq 0$ a unique solution exists and is given by[27]

$$\gamma = \frac{1}{\iota_k' A_k^{-1} \iota_k} \tag{37}$$

$$f_k = \frac{A_k^{-1} \iota_k}{\iota_k' A_k^{-1} \iota_k}. \tag{38}$$

If $f_k > 0$, we have a solution. If not, increase $k$ by 1 and repeat the procedure. The fact that $\underline{P} < 1/R_0$ yields another robustness check: the algorithm should terminate for some $k$ with $a_k < 1/R_0$. Otherwise, it fails to find the equilibrium.

Figure 10 gives an example for $\lambda = 0.4$ and $\mu = 0.2$. For these values, we can also use Proposition 4 to check the performance of our numerical procedure. With $l = 201$ grid points, our algorithm stops at $k = 78$ for an estimated lower bound of $\underline{P} = 0.53875$. The frequency distribution appears to consist of three different parts, with transitions around $0.67 \approx 1/R_0$ and $0.81 \approx \underline{P} R_0$.[28] This is also implied by Proposition 4. Figure 11 shows the corresponding CDF.

[27]To see this, note that we may first multiply (35) by $\iota_k' A_k^{-1}$ from the left (if $A_k$ is invertible), resulting in $\iota_k' f_k = \gamma \cdot \iota_k' A_k^{-1} \iota_k$. Substituting $\iota_k' f_k$ from (36) and dividing through $\iota_k' A_k^{-1} \iota_k$ yields (37). Plugging this back into $f_k = \gamma \cdot A_k^{-1} \iota_k$ (as obtained from (35)) gives $f_k$.

[28]The apparent discontinuity between the first and second price is an artifact of the discretization. It vanishes as the grid size $l$ increases.

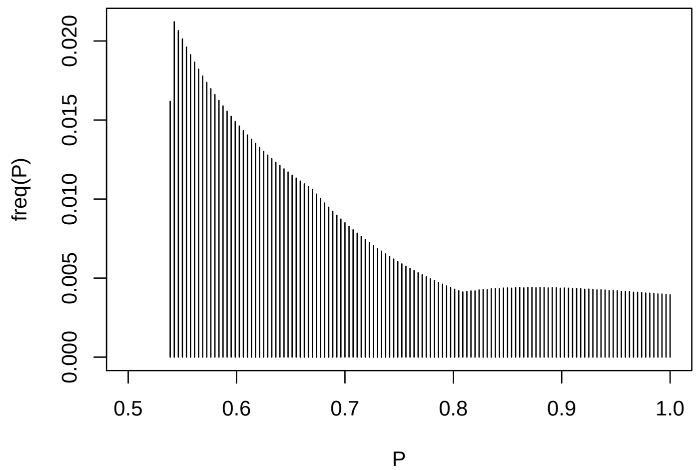



Figure 10: Approximated equilibrium PDF ($\lambda = 0.4$, $\mu = 0.2$).

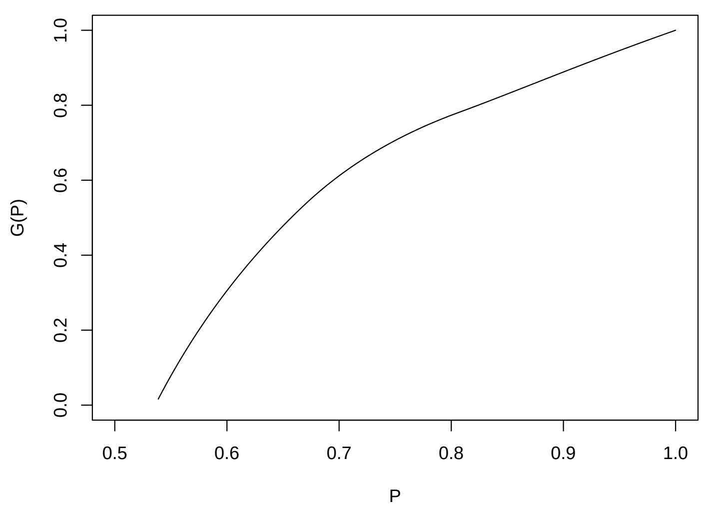



Figure 11: Approximated equilibrium CDF ($\lambda = 0.4$, $\mu = 0.2$).